Electrical Engineering and Computer Science
Syracuse University

Solutions to Homework #10

1. [10 points](L&D Exercise 10.2-1) For a DSB-SC system with a channel noise PSD of $S_n(f) = 10^{-12}$ and a baseband signal of bandwidth 5 kHz, the receiver output SNR is required to be at least 47 dB. The receiver is as shown in lecture notes.

(a) What must be the signal power S_i received at the receiver input?

(b) What is the receiver output noise power N_0?

Solution:

(a) $S_n(f) = \mathcal{N}2 = 10^{-12}$. Hence, $\mathcal{N} = 2 \times 10^{-12}$. Since the output SNR is required to be at least 47dB, then we have $\frac{S_o}{N_o} = 47dB = 50118$. Hence,

$$\frac{S_0}{N_0} = \frac{S_i}{\mathcal{N}B} = 50118,$$

which implies that

$$S_i = \mathcal{N}B \times 50118 = 2 \times 10^{-12} \times 5000 \times 50118 = 5.0118 \times 10^{-4}.$$

(b) The receiver output noise power $N_o = \mathcal{N}B = 2 \times 10^{-12} \times 5000 = 10^{-8}$.

2. [10 points](L&D Exercise 10.2-2) Repeat Problem 1 for SSB-SC.

Solution:

(a) Since the noise PSD is not changed, so $\mathcal{N}$ is the same as that in problem 1. The SNR of SSB is also the same as that of DSB-SC. Hence, the value of S_i is the same as that for problem 1, i.e., $S_i = 5.0118 \times 10^{-4}$.

(b) The receiver output noise power is also the same as that in problem 1, i.e., $N_o = 10^{-8}$.

3. [10 points](Adapted from L&D Exercise 10.2-3)Assume $\max_t m(t) = m_p$ in an AM system.

(a) Show that the output SNR for AM given in

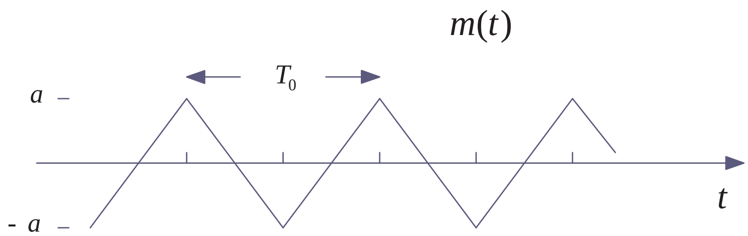
$$\frac{S_0}{N_0} = \frac{E[m^2]}{A^2 + E[m^2]}\gamma$$

can be expressed as

$$\frac{S_0}{N_0} = \frac{\mu^2}{k^2 + \mu^2}\gamma$$

where $k^2 = m_p^2/E[m^2]$ and μ is the modulation index.

(b) For the periodic triangle message signal shown below, find k^2. Suppose $\mu = 1$, express SNR in terms of γ and T_0.



Solution:
(a)

$$\frac{\mu^2}{k^2+\mu^2} = \frac{m_p^2/A^2}{m_p^2/E[m^2]+m_p^2/A^2} = \frac{E[m^2]}{A^2+E[m^2]}$$

So we can write

$$\frac{S_0}{N_0} = \frac{\mu^2}{k^2+\mu^2}\gamma$$

(b) We first compute

$$E[m^2] = \frac{1}{T_o}\left\{\int_0^{T_o/2}(\frac{4a}{T_o}t-a)^2dt + \int_{T_o/2}^{T_o}(-\frac{4a}{T_o}t+3a)^2dt\right\} = \frac{1}{3}a^2.$$

Then $k^2 = m_p^2/E[m^2] = 3$. Therefore, SNR is given by $\frac{S_0}{N_0} = \frac{\mu^2}{k^2+\mu^2}\gamma = \frac{1}{3+1}\gamma = \frac{1}{4}\gamma$.

4. [10 points] Consider a single tone signal $m(t) = \alpha\cos 2\pi f_m t$. Express S_0/N_0 for FM in terms of α, f_m, k_f, and γ.

Solution: The bandwidth of the baseband signal $m(t)$ is f_m, and $E[m^2] = \frac{1}{T_o}\int_0^{T_o}\alpha\cos 2\pi f_m t = \alpha^2/2$. So

$$\frac{S_o}{N_o} = 3\left(\frac{k_f^2E[m^2]}{(2\pi B)^2}\right)\gamma = \frac{3k_f^2\alpha^2\gamma}{2(2\pi f_m)^2}$$

5. [10 points] Consider the message signal $m(t)$ shown in figure in problem 3(b). Assume that the bandwidth of $m(t)$ is the frequency of its fifth harmonic frequency. Show that PM is superior to FM by a factor of approximately 20 from the SNR point of view, i.e., compute $\frac{\{S_0/N_0\}_{PM}}{\{S_0/N_0\}_{PM}}$.

Solution: Since the bandwidth of $m(t)$ is the frequency of its fifth harmonic frequency, $B = 5\times 1/T_o = 5/T_o$. Hence,

$$\frac{\{S_0/N_0\}_{PM}}{\{S_0/N_0\}_{PM}} = \frac{(2\pi B)^2m_p^2}{3\dot{m_p}^2} = \frac{(2\pi 5/T_o)^2a^2}{3(4a/T_o)^2} = \frac{25}{12}\pi^2 \approx 20$$