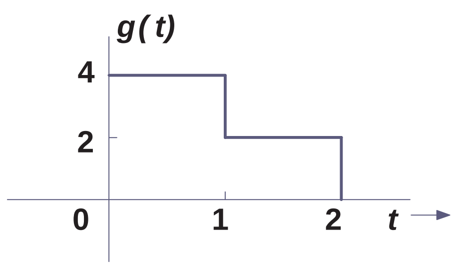
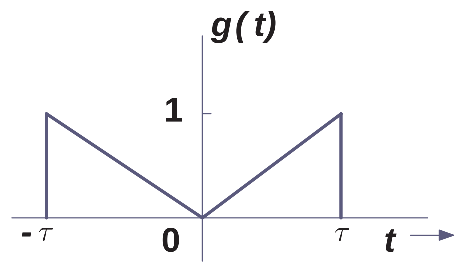
Electrical Engineering and Computer Science — ELE 551, Fall 2010
Syracuse University

Solutions to Homework #3

1. [10 points](L&D Exercise 3.1-4) Compute the Fourier transforms of the signals shown below.



(a)



(b)

Hint: The function $g(t)$ in (b) can be expressed as

$$g(t) = \begin{cases} \frac{t}{\tau} & 0 \leq t \leq \tau \\ -\frac{t}{\tau} & -\tau \leq t \leq 0 \\ 0 & \text{otherwise} \end{cases}$$

You may also want to use $\int te^{at}dt = \frac{e^{at}}{a^2}(at-1)$ for (b).

Solutions: (a)

$$\begin{aligned} G(f) &= \int_{-\infty}^{\infty} g(t)e^{-j2\pi ft}dt \\ &= \int_0^1 4e^{-j2\pi ft}dt + \int_1^2 2e^{-j2\pi ft}dt \\ &= \frac{4e^{-j2\pi ft}\Big|_0^1 + 2e^{-j2\pi ft}\Big|_1^2}{-j2\pi f} \\ &= \frac{4e^{-j2\pi f} - 4 + 2e^{-j4\pi f} - 2e^{-j2\pi f}}{-j2\pi f} \\ &= \frac{2e^{-j2\pi f} - 4 + 2e^{-j4\pi f}}{-j2\pi f} \\ &= \frac{4 - 2e^{-j2\pi f} - 2e^{-j4\pi f}}{j2\pi f} \end{aligned}$$

(b)

$$
\begin{aligned}
G(f) &= \int_{-\infty}^{\infty} g(t)e^{-j2\pi ft}dt \\
&= \int_{-\tau}^{0} -\frac{t}{\tau}e^{-j2\pi ft}dt + \int_{0}^{\tau} \frac{t}{\tau}e^{-j2\pi ft}dt \\
&= \frac{-e^{-j2\pi ft}(-j2\pi ft-1)\Big|_{-\tau}^{0} + e^{-j2\pi ft}(-j2\pi ft-1)\Big|_{0}^{\tau}}{\tau(-j2\pi f)^2} \\
&= \frac{1+e^{j2\pi f\tau}(j2\pi f\tau-1)+e^{-j2\pi f\tau}(-j2\pi f\tau-1)-(-1)}{\tau(-j2\pi f)^2} \\
&= \frac{2+j2\pi f\tau(e^{j2\pi f\tau}-e^{-j2\pi f\tau})-(e^{j2\pi f\tau}+e^{-j2\pi f\tau})}{-\tau(2\pi f)^2} \\
&= \frac{2-4\pi f\tau\sin 2\pi f\tau-2\cos 2\pi f\tau}{-\tau(2\pi f)^2} \\
&= \frac{4\pi f\tau\sin 2\pi f\tau+2\cos 2\pi f\tau-2}{\tau(2\pi f)^2}
\end{aligned}
$$

2. [10 points] (L&D Exercise 3.2-3) Show that

$$\sin(2\pi f_0 t+\theta) \Longleftrightarrow \frac{1}{2}\Big[\delta(f+f_0)e^{-j\theta+j0.5\pi}+\delta(f-f_0)e^{j\theta-j0.5\pi}\Big].$$

Hint: Use Euler's formula to express $\sin(2\pi f_0 t+\theta)$ in terms of exponentials.
Solution:

$$
\begin{aligned}
\sin(2\pi f_0 t+\theta) &= \frac{e^{j(2\pi f_0 t+\theta)}-e^{-j(2\pi f_0 t+\theta)}}{2j} \\
&= \frac{1}{2}\left[-je^{j2\pi f_0 t}e^{j\theta}+je^{-j2\pi f_0 t}e^{-j\theta}\right] \\
&= \frac{1}{2}\left[e^{j2\pi f_0 t}e^{j\theta-j0.5\pi}+e^{-j2\pi f_0 t}e^{-j\theta+j0.5\pi}\right]
\end{aligned}
$$

Since

$$e^{j2\pi f_0 t} \Longleftrightarrow \delta(f-f_0)$$

and

$$e^{-j2\pi f_0 t} \Longleftrightarrow \delta(f+f_0),$$

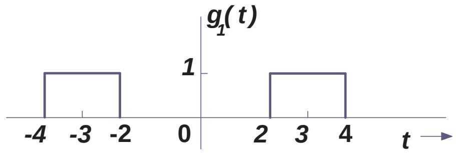
using linearity of Fourier transform, we obtain

$$\sin(2\pi f_0 t+\theta) \Longleftrightarrow \frac{1}{2}\Big[\delta(f-f_0)e^{j\theta-j0.5\pi}+\delta(f+f_0)e^{-j\theta+j0.5\pi}\Big].$$

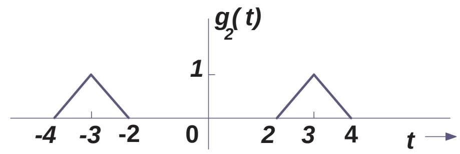
3. [10 points] (L&D Exercise 3.3-4) Use the time-shifting property to show that if $g(t) \Longleftrightarrow G(f)$, then

$$g(t+T)+g(t-T) \Longleftrightarrow 2G(f)\cos 2\pi fT.$$

Use this result and Fourier transforms for the rectangular and triangular functions to find the Fourier transforms of the signals shown below.



(a)



(b)

Hint: The functions $g_1(t)$ and $g_2(t)$ in the figure can be expressed as follows.

$$g_1(t) = \Pi\left(\frac{t-3}{2}\right) + \Pi\left(\frac{t+3}{2}\right)$$

$$g_2(t) = \triangle\left(\frac{t-3}{2}\right) + \triangle\left(\frac{t+3}{2}\right)$$

Solution:
By time-shifting and linearity properties,

$$g(t+T) + g(t-T) \Longleftrightarrow G(f)e^{j2\pi fT} + G(f)e^{-j2\pi fT} = 2G(f)\cos 2\pi fT.$$

Since $\Pi\left(\frac{t}{2}\right) \Longleftrightarrow 2\text{sinc}(2\pi f)$, we obtain

$$g_1(t) \Longleftrightarrow 2 \cdot 2\text{sinc}(2\pi f)\cos 2\pi f \cdot 3 = 4\text{sinc}(2\pi f)\cos 6\pi f.$$

Similarly, since $\triangle\left(\frac{t}{2}\right) \Longleftrightarrow \text{sinc}^2\left(\pi f\right)$, we obtain

$$g_2(t) \Longleftrightarrow 2\text{sinc}^2\left(\pi f\right)\cos 2\pi f \cdot 3 = 2\text{sinc}^2\left(\pi f\right)\cos 6\pi f.$$