Solutions to Homework 5

1. (a) The signal $m(t) = 40t$ for $-0.05 \le t \le 0.05$

$$P_m = \frac{1}{0.1}\int_{-0.05}^{0.05}(40t)^2 dt$$

$$= \frac{1}{0.1}\, 1600 \frac{t^3}{3}\Big|_{-0.05}^{0.05}$$

$$= \frac{1600}{0.3}\left(0.05^3 - (-0.05)^3\right)$$

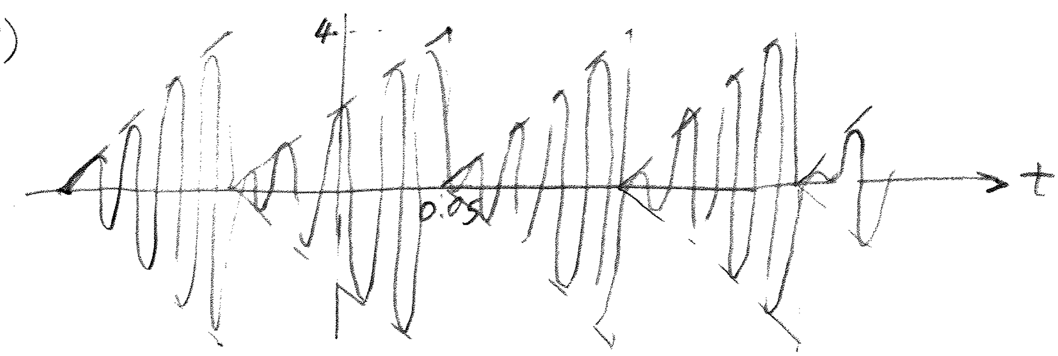
$$= \frac{4}{3} \text{ or } 1.33$$

(b) $m_p = 2$, $b = 2$

Modulation index $\mu = \frac{m_p}{b} = 1$

Power efficiency $\eta = \frac{P_m}{b^2 + P_m} = \frac{4/3}{4 + 4/3} = \frac{1}{4} = 25\%$.

(c)



2. (a) $\varphi_{AM}(t) = (A + m(t)) \cos 2\pi f_c t$

$$x(t) = \varphi_{AM}^2(t) = (A + m(t))^2 \cos^2 2\pi f_c t$$

$$= (A^2 + m^2(t) + 2Am(t))\, \frac{1 + \cos 4\pi f_c t}{2}$$

$$= \frac{1}{2}(A^2 + m^2(t) + 2Am(t)) + \frac{1}{2}(A^2 + m^2(t) + 2Am(t)) \cos 4\pi f_c t$$

Low-pass filter suppresses the high frequency term in $x(t)$, hence

$$y_1(t) = \frac{1}{2}(A^2 + m^2(t) + 2Am(t))$$

DC block suppresses the constant term in $y_1(t)$, hence.

$$y_2(t) = \frac{1}{2}\left(m^2(t) + 2Am(t)\right)$$

(b) In (a) $y_1(t) = \frac{1}{2}\left(A^2 + m^2(t) + 2Am(t)\right)$

$$= \frac{A^2}{2}\left(1 + \left(\frac{m(t)}{A}\right)^2 + 2\frac{m(t)}{A}\right)$$

If $A >> m(t)$ for all t, then

$$y_1(t) \approx \frac{A^2}{2}\left(1 + 2\frac{m(t)}{A}\right)$$

$$= \frac{A^2}{2} + Am(t)$$

Now, $y_2(t) = Am(t)$. Thus, $y_2(t)$ recovers $m(t)$.

3. (a) $\left[m_1(t)\cos w_c t + m_2(t)\sin w_c t\right]\cos\left((w_c + \Delta w)t + \delta\right)$

$$= m_1(t)\cos w_c t\cos\left((w_c + \Delta w)t + \delta\right) + m_2(t)\sin w_c t\cos\left((w_c + \Delta w)t + \delta\right)$$

$$= \frac{1}{2}m_1(t)\cos(\Delta wt + \delta) + \frac{1}{2}m_1(t)\cos\left((2w_c t + \Delta w)t + \delta\right)$$

$$- \frac{1}{2}m_2(t)\sin(\Delta wt + \delta) + \frac{1}{2}m_2(t)\sin\left((2w_c t + \Delta w)t + \delta\right)$$

The low-pass filter suppresses the high frequency terms, then the output now is.

$$\frac{1}{2}m_1(t)\cos(\Delta wt + \delta) - \frac{1}{2}m_2(t)\sin(\Delta wt + \delta)$$

(b) $\left[m_1(t)\cos w_c t + m_2(t)\sin w_c t\right]\sin\left[(w_c + \Delta w)t + \delta\right]$

$$= m_1(t)\cos w_c t\sin\left[(w_c + \Delta w)t + \delta\right] + m_2(t)\sin w_c t\sin\left[(w_c + \Delta w)t + \delta\right]$$

$$= \frac{1}{2}m_1(t)\sin(\Delta wt + \delta) + \frac{1}{2}m_1(t)\sin\left((2w_c + \Delta w)t + \delta\right)$$

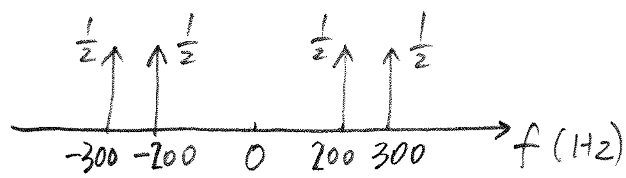
$$+ \frac{1}{2}m_2(t)\cos(\Delta wt + \delta) - \frac{1}{2}m_2(t)\cos\left((2w_c + \Delta w)t + \delta\right)$$

The low-pass filter suppress the high frequency terms, then the output is

$$\frac{1}{2} m_1(t) \sin(\Delta\omega t + \delta) + \frac{1}{2} m_2(t) \cos(\Delta\omega t + \delta)$$

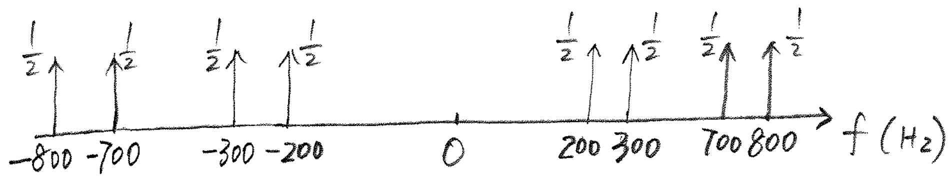
4. (a) $m(t) = 2\cos 100\pi t \cos 500\pi t$

$$= \cos 400\pi t + \cos 600\pi t$$

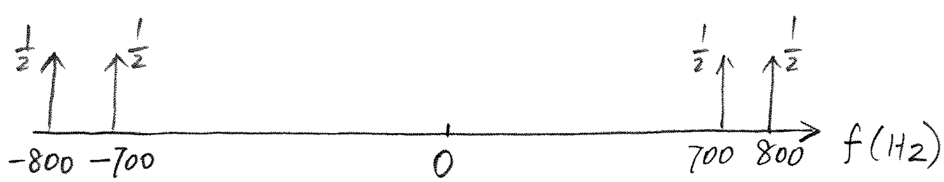


(b) $2m(t) \cos 1000\pi t$

DSB-SC has a spectrum which shifts spectrum of $m(t)$ to the left and right by 500 Hz



(c) USB.

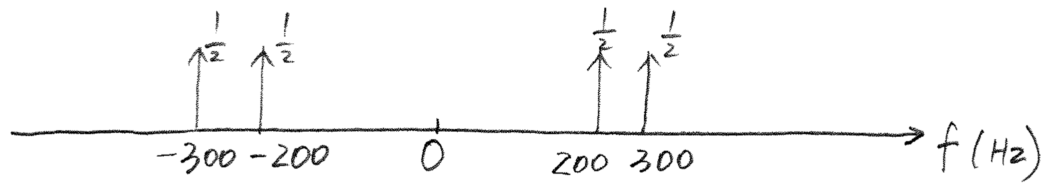


$$\Phi_{USB}(f) = \frac{1}{2}\delta(f-700) + \frac{1}{2}\delta(f+700) + \frac{1}{2}\delta(f-800) + \frac{1}{2}\delta(f+800)$$

Hence $\varphi_{USB}(t) = \frac{1}{2}e^{j1400\pi t} + \frac{1}{2}e^{-j1400\pi t} + \frac{1}{2}e^{j1600\pi t} + \frac{1}{2}e^{-j1600\pi t}$

$$= \cos 1400\pi t + \cos 1600\pi t$$

(d) LSB.



$$\Phi_{LSB}(f) = \frac{1}{2}\delta(f-200) + \frac{1}{2}\delta(f+200) + \frac{1}{2}\delta(f-300) + \frac{1}{2}\delta(f+300)$$

Hence $\varphi_{LSB}(t) = \frac{1}{2}e^{j400\pi t} + \frac{1}{2}e^{-j400\pi t} + \frac{1}{2}e^{+j600\pi t} + \frac{1}{2}e^{-j600\pi t}$

$$= \cos 400\pi t + \cos 600\pi t$$

5. (a) $m(t) = 2\cos 100\pi t \cos 500\pi t$

$$= \cos 600\pi t + \cos 400\pi t$$

Use hint:

$$m_h(t) = \cos\left(600\pi t - \frac{\pi}{2}\right) + \cos\left(400\pi t - \frac{\pi}{2}\right)$$

$$= \sin 600\pi t + \sin 400\pi t$$

(b) $\varphi_{USB}(t) = m(t)\cos 1000\pi t - m_h \sin 1000\pi t$

$$= (\cos 600\pi t + \cos 400\pi t)\cos 1000\pi t - (\sin 600\pi t + \sin 400\pi t)\sin 1000\pi t$$

$$= \cos 600\pi t \cos 1000\pi t + \cos 400\pi t \cos 1000\pi t$$

$$- \sin 600\pi t \sin 1000\pi t - \sin 400\pi t \sin 1000\pi t$$

$$= \frac{1}{2}\cos 400\pi t + \frac{1}{2}\cos 1600\pi t + \frac{1}{2}\cos 600\pi t + \frac{1}{2}\cos 1400\pi t$$

$$- \frac{1}{2}\cos 400\pi t + \frac{1}{2}\cos 1600\pi t - \frac{1}{2}\cos 600\pi t + \frac{1}{2}\cos 1400\pi t$$

$$= \cos 1600\pi t + \cos 1400\pi t$$

$$\varphi_{LSB}(t) = m(t)\cos 1000\pi t + m_h(t)\sin 1000\pi t$$

$$= (\cos 600\pi t + \cos 400\pi t)\cos 1000\pi t + (\sin 600\pi t + \sin 400\pi t)\sin 1000\pi t$$

$$= \cos 600\pi t \cos 1000\pi t + \cos 400\pi t \cos 1000\pi t$$

$$+ \sin 600\pi t \sin 1000\pi t + \sin 400\pi t \sin 1000\pi t$$

$$= \frac{1}{2}\cos 400\pi t + \frac{1}{2}\cos 1600\pi t + \frac{1}{2}\cos 600\pi t + \frac{1}{2}\cos 1400\pi t$$

$$+ \frac{1}{2}\cos 400\pi t - \frac{1}{2}\cos 1600\pi t + \frac{1}{2}\cos 600\pi t - \frac{1}{2}\cos 1400\pi t$$

$$= \cos 400\pi t + \cos 600\pi t$$

$\varphi_{USB}(t)$ and $\varphi_{LSB}(t)$ are the same as those in 4(c) and 4(d)!