Electrical Engineering and Computer Science
Syracuse University

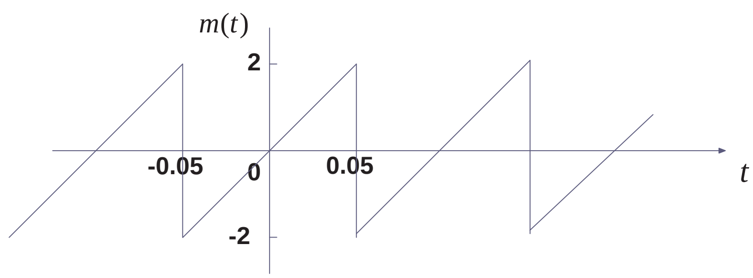
ELE 551, Fall 2010

Homework #5

Due on **October 21, 2010, Thursday** In Class

1. [10 points] (Adapted from L&D Exercise 4.3-1) In an amplitude modulation system, a periodic message signal is given by the following figure and the carrier frequency is 1 kHz. The modulator output is

$$s_{AM}(t) = [b + m(t)] \cos w_c t.$$

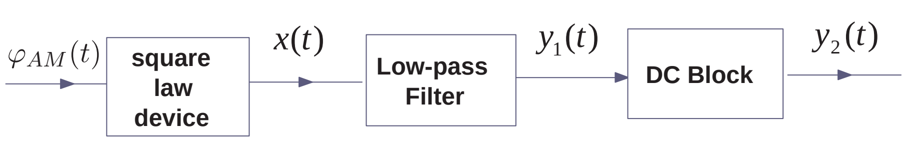


(a) Determine the average power of $m(t)$ (compute the numerical value).
(b) If $b = 2$, determine the modulation index and the modulation power efficiency.
(c) Sketch the modulated signal of part (a) in the time domain.

2. [10 points] (Adapted from L&D Exercise 4.3-8) In the early days of radio, AM signals were demodulated by a crystal detector followed by a low-pass filter and a dc blocker, as shown in the following figure. Assume the modulated signal is

$$\varphi_{AM}(t) = (A + m(t)) \cos 2\pi f_c t.$$

Assume a crystal detector is basically a squaring device so that $x(t) = [\varphi_{AM}(t)]^2$. The low pass filter suppress the high frequency spectrum. The DC block suppress the constant terms (i.e., the terms that are not time-varying) in the signal.



(a) Determining the signals $x(t)$, $y_1(t)$ and $y_2(t)$ at time domain.
(b) If $A \gg m(t)$ for all t, what approximation can you obtain for $y_1(t)$? Does the signal $y_2(t)$ now recover $m(t)$?

3. [10 points] (Adapted from L&D Exercise 4.4-1) Consider the demodulation of a QAM signal $m_1(t) \cos w_c t + m_2(t) \sin w_c t$.

(a) Suppose the receiver uses the carrier $\cos[(w_c + \Delta w)t + \delta]$ to recover $m_1(t)$. Show that the output of the receiver after passing through the low-pass filter is given by

$$m_1(t)\cos[(\Delta w)t + \delta] - m_2(t)\sin[(\Delta w)t + \delta]$$

in stead of $m_1(t)$.

(b) Suppose the receiver uses $\sin[(w_c + \Delta w)t + \delta]$ to recover $m_2(t)$. Show that the output of the receiver after passing through the low-pass filter is given by

$$m_1(t)\sin[(\Delta w)t + \delta] + m_2(t)\cos[(\Delta w)t + \delta]$$

in stead of $m_2(t)$.

4. [10 points] (Partially adapted from L&D Exercise 4.4-2) A modulating signal $m(t)$ is given by

$$m(t) = 2\cos 100\pi t \cos 500\pi t$$

(a) Sketch the spectrum of $m(t)$. (Hint: Express $m(t)$ in terms of addition of cosine functions.)

(b) Find and sketch the spectrum of the DSB-SC signal $2m(t)\cos 1000\pi t$.

(c) From the spectrum obtained in part (b), suppress the LSB spectrum to obtain the USB spectrum. Write out the expression $\varphi_{USB}(t)$ for the USB signal.

(d) From the spectrum obtained in part (b), suppress the USB spectrum to obtain the LSB spectrum. Write out the expression $\varphi_{LSB}(t)$ for the LSB signal.

5. [10 points] (Adapted from L&D Exercise 4.4-3) For the same signal as in problem 4,

$$m(t) = 2\cos 100\pi t \cos 500\pi t$$

(a) Find the Hilbert transform $m_h(t)$ of the signal $m(t)$. (Hint: (1) Express $m(t)$ in terms of addition of cosine functions. (2) If $m(t)$ is a sinusoid, its Hilbert transform $m_h(t)$ is the $m(t)$ phased delayed by $\pi/2$ rad.)

(b) Use the following equations ($f_c = 500$):

$$\varphi_{USB} = m(t)\cos 2\pi f_c t - m_h(t)\sin 2\pi f_c t$$
$$\varphi_{LSB} = m(t)\cos 2\pi f_c t + m_h(t)\sin 2\pi f_c t$$

to compute the USB and LSB signals. Are your answers the same as those in 4 (c) and 4 (d)?