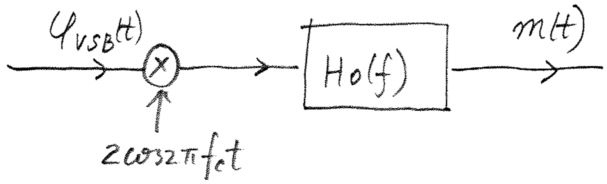
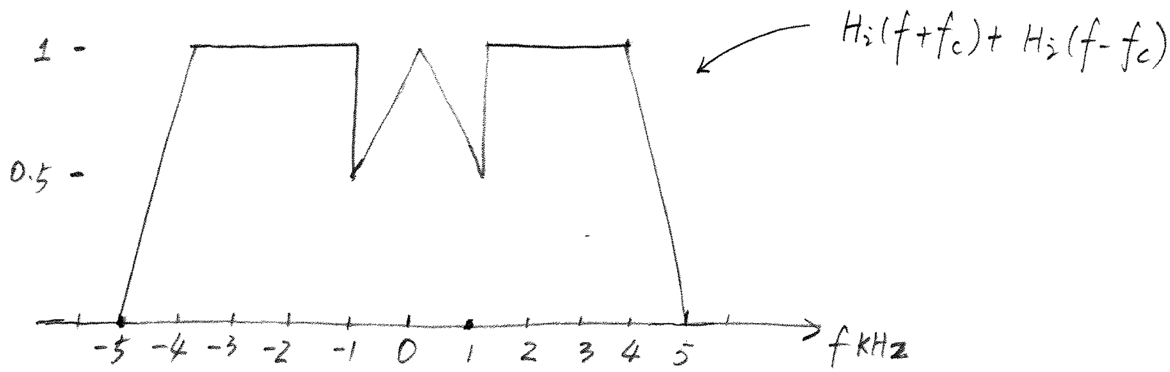
Solutions to Homework 6

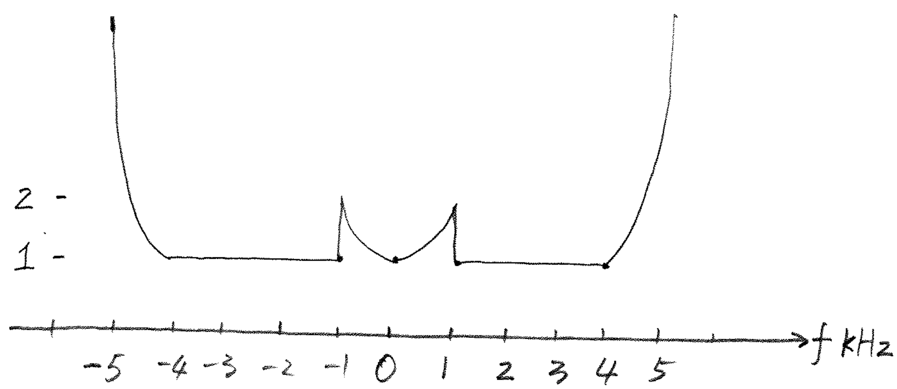
1. (a)



(b) $B = (1501 - 1496) = 5\,kHz$.

(c) $$H_0(f) = \frac{1}{H_i(f+f_c) + H_i(f-f_c)}$$





2. $\omega_c = 10^8$, $k_f = 10^5$, $k_p = 25$

FM: The instantaneous frequency is $\omega_i = 10^8 + 10^5 m(t)$, $(\omega_i)_{\min} = 10^8 - 10^5 = 9.99 \times 10^7$ rad/s, $(\omega_i)_{\max} = 10^8 + 10^5 = 1.001 \times 10^8$ rad/s. Figure S5.1-1 shows that the cycle is split into four equal parts of length $10^{-3}/4 = 2.5 \times 10^{-4}$ s. The instantaneous frequency increases linearly from $(\omega_i)_{\min}$ to $(\omega_i)_{\max}$, stays there, then decreases linearly back to $(\omega_i)_{\min}$, and stays there for the last quarter-cycle.

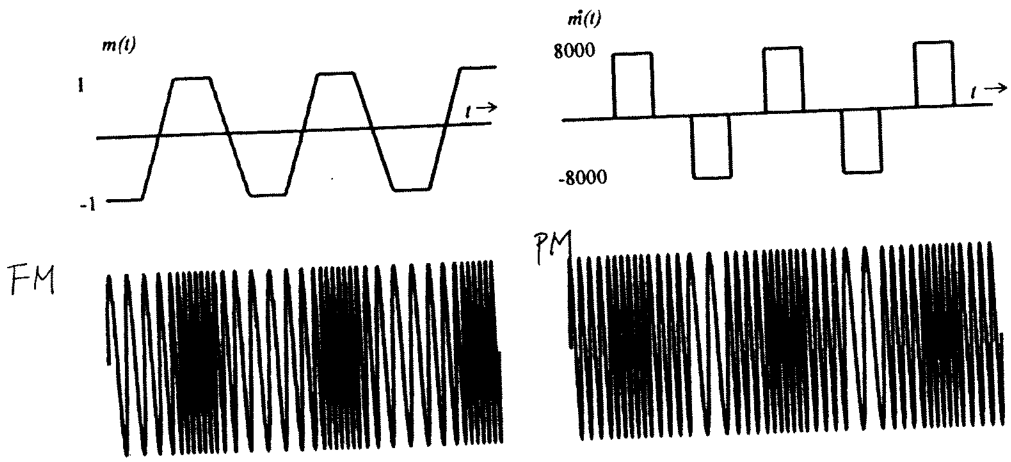


Fig. S5.1-1

PM: In this case $\omega_i = 10^8 + 25\dot{m}(t)$ with $(\dot{m}(t))_{\max} = 8000$, $(\dot{m}(t))_{\min} = -8000$, $(\omega_i)_{\min} = 10^8 - 2 \times 10^5 = 9.98 \times 10^7$ rad/s, $(\omega_i)_{\max} = 10^8 + 2 \times 10^5 = 1.002 \times 10^8$ rad/s. Figure S5.1-1 shows that the waveform, in this case, stays at $(\omega_i)_{\max}$ for one quarter-cycle, shifts to ω_c, shifts to $(\omega_i)_{\min}$, and then shifts back to ω_c for the last quarter cycle.

3.

(a) $\omega_c = 2\pi \times 10^6$, $k_f = 2000\pi$, $k_p = \pi/2$.

FM: The instantaneous frequency is $f_i = 10^6 + 1000m(t)$, $(f_i)_{\min} = 10^6 - 1000 = 999$ kHz, $(f_i)_{\max} = 10^6 + 1000 = 1001$ kHz. As shown in Fig. S5.1-2 the instantaneous frequency of the FM wave increases linearly from 999 to 1001 kHz over 10^{-3} s, then switches back to 999 kHz and repeats.

PM: Since $m(t)$ has jump discontinuities, the direct approach will be used. When one cycle of the sawtooth is centered on the origin, $m(t) = 2000t$ over that cycle. Hence,

$$
\begin{aligned}
\varphi_{\text{PM}}(t) &= \cos\left[2\pi \times 10^6 t + \frac{\pi}{2} m(t)\right] \\
&= \cos\left[2\pi \times 10^6 t + \frac{\pi}{2} 2000t\right] \\
&= \cos\left[2\pi\left(10^6 + 500\right)t\right].
\end{aligned}
$$

As shown in Fig. S5.1-2, at the discontinuity there is a jump of $2k_p = \pi$, otherwise, the carrier frequency is constant at $10^6 + 500$ Hz.

(b) This is equivalent to another PM signal with $f_c = 1000.5$ kHz and periodic rectangular message that switches from 1 to -1, as shown in the example at the beginning of the chapter. It is necessary to keep k_p less than π, since those periodic signal jumps at those points of discontinuity $\Delta = 2$; otherwise, a larger k_p will give rise to phase ambiguity when $k_p\Delta > 2\pi$.

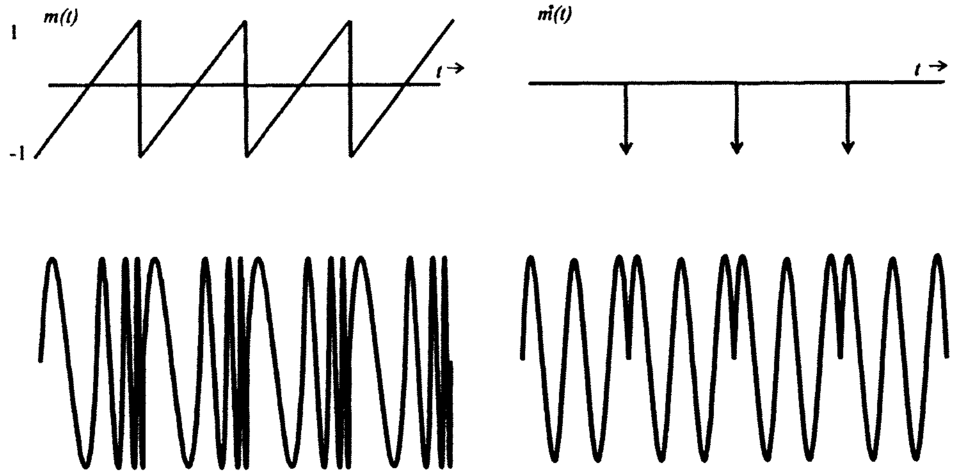


Fig. S5.1-2

4. We are given $\omega_c = 10000\pi$, and that over $|t| \leq 1$,

$$\varphi_{\text{EM}}(t) = 10\cos 13{,}000\pi t$$

(a) If this were a PM signal with $k_p = 1000$, we would have

$$\begin{aligned}\varphi_{PM}(t) &= 10\cos 13000\pi t = 10\cos\left[\omega_c t + k_p m(t)\right] \\ &= 10\cos\left[10000\pi t + 1000 m(t)\right]\end{aligned}$$

Clearly, $m(t) = 3\pi t$ over this interval.

(b) For an FM signal with $k_f = 1000$,

$$\varphi_{FM}(t) = A\cos\left[\omega_c t + k_f\int_0^t m(\alpha)\,d\alpha\right] = 10\cos\left[10{,}000\pi t + 1000\int_0^t m(\alpha)d\alpha\right].$$

Therefore,

$$\int_0^t m(\alpha)d\alpha = 3\pi t$$

and $m(t) = 3\pi$ over the interval.