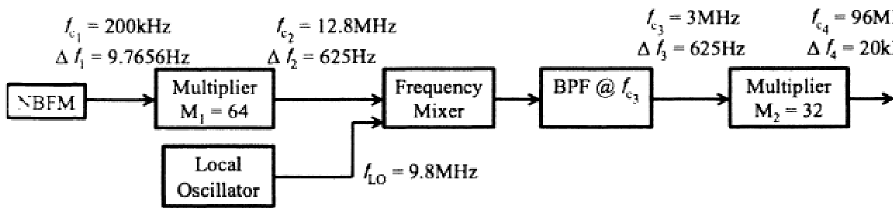
Electrical Engineering and Computer Science ELE 551, Fall 2010
Syracuse University

Solutions to Homework #8

1. [10 points](L&D Exercise 5.3-1) Design (the block diagram of) an Armstrong indirect FM modulator to generate an FM carrier with a carrier frequency of 96 MHz and $\Delta f = 20$ kHz. A narrowband FM generator with $f_c = 200$ kHz and adjustable Δf in the range of 9 to 10 Hz is available. The stockroom also has an oscillator with adjustable frequency in the range of 9 to 10 MHz. There are bandpass filters with any center frequency, and only frequency doublers are available.

Solutions: The design is shown in the figure below.



In this case, the NBFM generator generates $f_{c_1} = 200kHz$ and $\Delta f_1 = 9$ to $10Hz$. The final WBFM should have $f_{c4} = 96MHz$ and $\Delta f_4 = 20kHz$. The total factor of frequency multiplication needed is $M_1 \cdot M_2 = \frac{\Delta f_4}{\Delta f_1} = 2000$ to 2222. Because only frequency doublers are available, we find that $M_1 \cdot M_2 = 2^{11}$, and $\Delta f_1 = \frac{\Delta f_4}{2^{11}} = 9.7656Hz$.

Now, let $M_1 = 2^{n_1}$, $M_2 = 2^{n_2}$, $n_1 + n_2 = 11$, $f_{c_2} = 2^{n_1} f_{c_1}$, and $f_{c_4} = 2^{n_2} f_{c_3}$. In order to find f_{LO}, there are three possible relationships: $f_{c_3} = f_{c_2} \pm f_{LO}$ and $f_{LO} - f_{c_2}$. Each should be tested to determine the one that will require $9^6 \leq f_{LO} \leq 10^7$.

First, we test $f_{c_3} = f_{c_2} - f_{LO}$. This case leads to

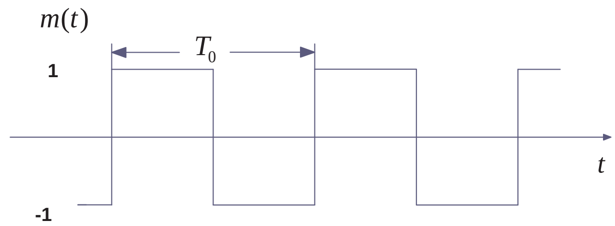
$$96MHz = f_{c_4} = 2^{n_2} f_{c_3} = 2^{n_2}(f_{c_2} - f_{LO}) = 2^{n_2}(2^{n_1} f_{c_1} - f_{LO}) = 2^{11}(200 \times 10^3) - 2^{n_2} f_{LO}.$$

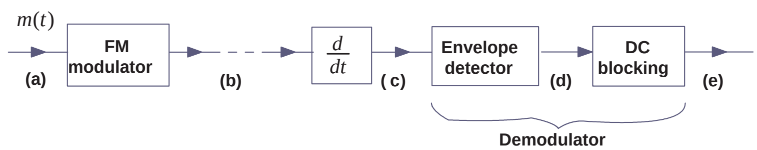
Thus, we have

$$f_{LO} = 2^{-n_2}(4.096 \times 10^8 - 9.6 \times 10^7) = 2^{-n_2}(3.136 \times 10^8).$$

In this case, if $n_2 = 5$ then $f_{LO} = 9.8$MHz, which is in the desired range. We will not test the other cases since this one works. Thus, the final design is $M_1 = 2^6 = 64$, $M_2 = 32$, and $f_{LO} = 9.8$MHz. This gives $f_{c_2} = 2^{n_1} f_{c_1} = 12.8$MHz, $\Delta f_2 = M_1 \times \Delta f_1 = 625$Hz, $f_{c_3} = f_{c_2} - f_{LO} = 12.8 - 9.8 = 3$MHz, $\Delta f_3 = 625$Hz. The bandpass filter used will be centered at 3 MHz.

2. [10 points] (L&D Exercise 5.4-2) A periodic square wave $m(t)$ (shown in the figure below) frequency-modulates a carrier of frequency $f_c = 10$ kHz with $\Delta f = 1$ kHz. The carrier amplitude is A. The resulting FM signal is demodulated, as also shown in the figure below. Sketch the waveforms at points b, c, d, and e. (Note that the DC blocking in the following figure suppresses the none time-varying part of the signal.





Solutions: Given that $f_c = 10$ kHz, $\Delta f = 1$ kHz, and that the message is periodic square wave of period T_0, the resulting FM signal has instantaneous frequency $f_i = f_c \pm \Delta f$, which switches from 11 kHz to 9 kHz and back over one period. Thus,

$$\varphi_{FM}(t) = A\cos[20000\pi t \pm 2000\pi t]$$

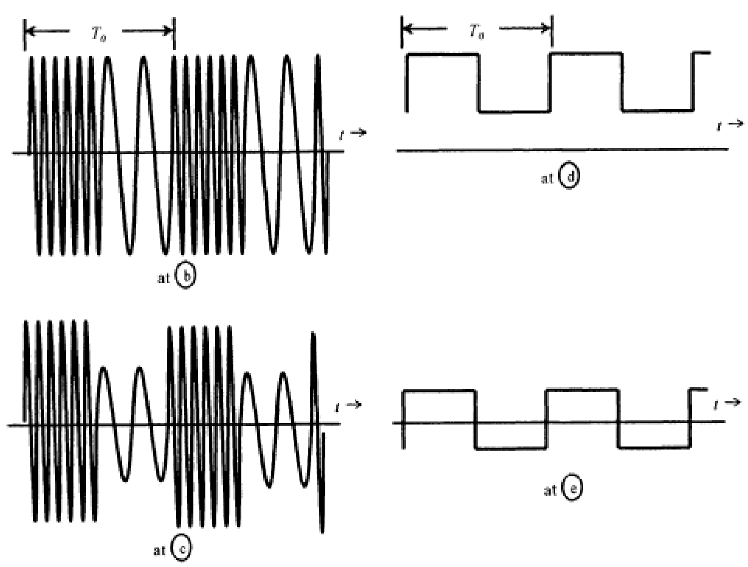
over any given half period. This signal is shown in the figure below as (b).

Now, after the ideal differentiator,

$$\dot{\varphi}_{FM}(t) = -(20000\pi \pm 2000\pi)A\sin[20000\pi t \pm 2000\pi t]$$

This signal is shown in the figure below as (c). Comparing with (b), this signal also has its amplitude switching between $22000\pi A$ and $18000\pi A$.

Next, after the envelope detector, the output will be a periodic square wave proportional to $(20000\pi \pm 2000\pi)A$, with a DC offset as shown in the figure below as (d). After DC blocking, the DC constant 20000π is removed, and the result is a periodic square wave proportional to $m(t)$. This is illustrated in the figure below as (e).



3. [10 points] (L&D Exercise 5.4-3) Let $s(t)$ be an angle-modulated signal that a receiver obtains,

$$s(t) = 2\cos\left[10^7\pi t + 2\sin(2000\pi t + 0.3\pi) - 3\pi\cos(100t)\right].$$

(a) Find the bandwidth of this FM signal.

(b) If $s(t)$ is sent to an (ideal) envelope detector, find the detector output signal.

(c) If $s(t)$ is first differentiated before the envelope detector, find the detector output signal.

(d) Explain which detector output can be processed to yield the message signal $m(t)$ and find the message signal $m(t)$ if $k_f = 200\pi$.

Solutions:

(a) From the form of $s(t)$, we know that

$$k_f \int_0^t m(\alpha)d\alpha = 2\sin(2000\pi t + 0.3\pi) - 3\pi\cos(100t),$$

Taking derivative of the above equation, we have

$$k_f m(t) = 4000\pi\cos(2000\pi t + 0.3\pi) + 300\pi\sin(100t),$$

It is clear that the bandwidth of $m(t)$ is $B = 1$kHz. From the above equation, we also have $k_f m_p = 4300\pi$ and $\Delta f = \frac{k_f m_p}{2\pi} = 2150$Hz. Therefore, $B_{FM} = 2(\Delta f + B) = 6.3$kHz.

(b) The envelope of $s(t)$ is 2.

(c) Taking derivative of $s(t)$ with respect to t, we get

$$\dot{s}(t) = -2(10^7\pi + 4000\pi\cos(2000\pi t + 0.3\pi) + 300\pi\sin(100t))\sin(10^7\pi t + 2\sin(2000\pi t + 0.3\pi) - 3\pi\cos(100t)),$$

so the envelope detector output is

$$2(10^7\pi + 4000\pi\cos(2000\pi t + 0.3\pi) + 300\pi\sin(100t)).$$

(d) The envelope in (c) can be processed to yield the message signal $m(t)$. Passing the envelope of $\dot{s}(t)$ through a DC blocking, the signal becomes $8000\pi\cos(2000\pi t + 0.3\pi) + 600\pi\sin(100t))$, which is $2k_f m(t)$. We then pass this signal through an amplifier with gain $\frac{1}{2k_f} = \frac{1}{400\pi}$ to obtain the output $m(t)$.