Electrical Engineering and Computer Science
Syracuse University

ELE 551, Fall 2010

Homework #9

Due on November 30 Tuesday, in Class

1. [10 points](L&D Exercise 9.1-7) Consider the random process

$$x(t) = at^2 + b$$

where b is a constant and a is a random variable uniformly distributed in the range $(-2, 2)$, i.e., the pdf of a is given by

$$p_a(x) = \begin{cases} \frac{1}{4} & -2 < a < 2 \\ 0 & otherwise \end{cases}.$$

(a) Find the mean of $x(t)$, i.e., $E[x(t)]$.
(b) Find the autocorrelation function $x(t)$, i.e., $R_x(t_1, t_2) = E[x(t_1)x(t_2)]$.
(c) Determine whether $x(t)$ is a wide-sense stationary process or not.

2. [10 points] Let $x(t) = kt$, where k is a random variable uniformly distributed over $(-1, 1)$, i.e., the pdf of k is given by

$$p_k(x) = \begin{cases} \frac{1}{2} & -1 < x < 1 \\ 0 & otherwise \end{cases}.$$

(a) Find the mean of $x(t)$, i.e., $E[x(t)]$.
(b) Find the autocorrelation function $x(t)$, i.e., $R_x(t_1, t_2) = E[x(t_1)x(t_2)]$.
(c) Determine whether $x(t)$ is a wide-sense stationary process or not.

3. [10 points] Suppose random process $y(t)$ is the output of a linear time-invariant system with WSS input $x(t)$. The transfer function of the linear system is given by $H(f)$. Indicate whether the following statements are true or false. Justify your answers.
(a) The output $y(t)$ is a WSS process.
(b) If $H(f) \leq 1$ for all f, then the power of $y(t)$ is less than or equal to the power of $x(t)$.

4. [10 points] A white noise process of PSD $\mathcal{N}/2$ is transmitted through a bandpass filter $H(f)$ (shown in the figure below). Assume the center frequency used in this representation is 100 kHz.
(a) Represent the filter output $n(t)$ in terms of quadrature components, and determine $S_{n_c}(f)$ and $S_{n_s}(f)$.
(b) Find $E[n_c^2], E[n_s^2]$, and $E[n^2]$.