Electrical Engineering and Computer Science
Syracuse University

ELE 551, Fall 2010

Midterm Exam I

On **October 7, 2010, 2:00pm-3:20pm**

You have 80 minutes to complete the exam. You may take one paper-size one-sided notes. No other notes, books, and calculators are allowed. Show your work for full credit.

Your Name:__________________

Questions	Total Points	Your Score
1	10	
2	10	
3	10	
4	10	
5	20	
6	20	
Total	**80**	

Some Useful Information:

- Fourier Transform:

$$\begin{aligned} e^{-at}u(t) &\Longleftrightarrow \frac{1}{a+j2\pi f} \\ e^{j2\pi f_0 t} &\Longleftrightarrow \delta(f-f_0) \\ B\text{sinc}^2(\pi B t) &\Longleftrightarrow \triangle\left(\frac{f}{2B}\right) \\ \delta(t) &\Longleftrightarrow 1 \\ \cos 2\pi f_0 t &\Longleftrightarrow \frac{1}{2}\left[\delta(f+f_0)+\delta(f-f_0)\right] \end{aligned}$$

- Properties of Fourier Transform
  - Frequency shifting: $g(t)e^{j2\pi f_0 t} \Leftrightarrow G(f-f_0)$
  - Time shifting: $g(t-t_0) \Leftrightarrow G(f)e^{-j2\pi f t_0}$
- $\int e^{ax}dx = \frac{1}{a}e^{ax}$
- Trigonometric Identities:

$$\begin{aligned} &\cos x = \frac{e^{jx}+e^{-jx}}{2} \\ &\sin x = \frac{e^{jx}-e^{-jx}}{2j} \\ &\sin\left(x \pm \frac{\pi}{2}\right) = \pm\cos x \\ &\sin^2 x = \frac{1}{2}(1-\cos 2x) \\ &\cos^2 x = \frac{1}{2}(1+\cos 2x) \\ &\sin\frac{\pi}{2} = 1 \\ &\cos x \cos y = \frac{1}{2}[\cos(x-y)+\cos(x+y)] \end{aligned}$$

1. [10pt] True or False. Circle “True” if the statement is correct, and circle “False” if the statement is not correct. Justification is NOT needed.

(a) If $g(t)$ is a periodic signal with period T_0, its average power can be computed as $P_g = \frac{1}{T_0}\int_{T_0} g^2(t)dt$.

True False

(b) A periodic signal can be expressed by a trigonometric Fourier series, but can NOT be expressed by exponential Fourier series.

True False

(c) If $g(t)$ is a real function of t, then $G(-f) = G^*(f)$.

True False

(d) If the LTI system has frequency response $H(f)$. Then energy spectral density (ESD) of its input and output has the relationship $\Psi_y(f) = |H(f)|^2\Psi_x(f)$.

True False

(e) Consider a LTI system that maps any input $x(t)$ to an output $y(t) = x(t) + x(t - T)$, where T is a delay constant. Then the system response is given by $h(t) = \delta(t) + \delta(t - T)$.

True False

2. [10pt] Find the Fourier transform of the signal $g(t) = e^{-t}u(t)\sin 2\pi f_c t$, where $u(t)$ is the step function. (Hint: Express $\sin 2\pi f_c t$ in terms of exponentials. You may directly use the Fourier transform for the function $e^{-t}u(t)$, and the frequency shifting property.)

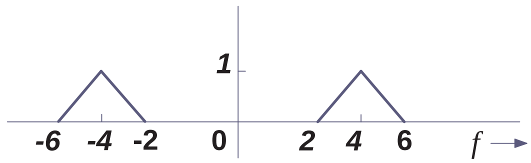
3. [10pt] Find the inverse Fourier transform of $G(f)$ given below by Direct Calculation.

$$G(f) = \begin{cases} e^{j\pi/2}, & -B < f < 0 \\ e^{-j\pi/2}, & 0 \leq f < B \\ 0, & \text{otherwise} \end{cases}$$

4. [10pt] Suppose $G(f)$ is the Fourier transform of $g(t)$.

(a) What is the inverse Fourier transform of $G(f - f_0) + G(f + f_0)$? (Hint: Use frequency shifting property.)

(b) What is the inverse Fourier transform of the following frequency domain representation?



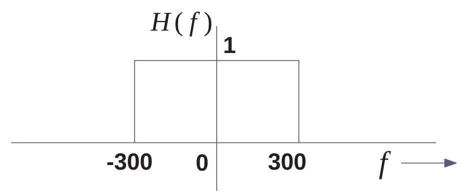
5. [20pt] Consider a baseband signal $m(t) = \cos 500\pi t$. In your plots, please mark the amplitude of each impulse.

(a) Plot the spectrum of $m(t)$.

(b) Plot the spectrum of the DSB-SC signal $2m(t)\cos 4500\pi t$.

(c) Now suppose the DSB-SC signal in (b) is multiplied by a sinusoidal signal $2\cos 5000\pi t$. Sketch the spectrum now.

(d) Suppose the signal in (c) is passed through the following low pass filter centered at origin with bandwidth 300 Hz. Sketch the spectrum now.



(e) In (d), you do not recover $m(t)$. Why?

6. [20pt] The baseband signal $m(t)$ whose spectrum M(f) is shown in the figure is passed through the system shown in the same figure. The frequency response of $H_1(f)$ and $H_2(f)$ are also given below. Assume that $f_0 >> W$.

(a) Plot the spectrum of $x(t)$.

(b) Show that

$$y_1(t) = m^2(t) + \frac{1}{2} + \frac{1}{2}\cos 4\pi f_0 t + 2m(t)\cos 2\pi f_0 t.$$

(c) Plot the spectrum of $y_1(t)$. (Hint: Find Fourier transform for each term of $y_1(t)$ given in (b). The spectrum of $m^2(t) + \frac{1}{2}$ is given in the figure below.)

(d) Plot the spectrum of $y_2(t)$.

(e) Plot the spectrum of $y_3(t)$.

(f) Plot the spectrum of $y_4(t)$.