Solutions to Midterm II

1. [10pt] True or False. Circle "True" if the statement is correct, and circle "False" if the statement is not correct. Justification is NOT needed.

(a) Any AM signal can be demodulated by envelope detector.

True False

(b) In quadrature amplitude modulation (QAM), two signals occupy the same band, and hence its usage of bandwidth is twice as efficient as DSB-SC.

True False

(c) AM modulated signals have time varying amplitude but fixed carrier frequency. However, FM signals have constant amplitude but time varying frequency.

True False

(d) In generating wideband FM signals, the frequency multiplier increases the frequency deviation, but keeps the carrier frequency unchanged.

True False

(e) The phase locked loop (PLL) can be used to track the phase of an incoming signal. However, there may be a constant error between incoming and output signal phases.

True False

2. [10pt] A DSB-SC modulated signal $m(t)\cos 2\pi f_c t$ is demodulated by multiplying it with a carrier signal $2\cos 2\pi(f_c + \Delta f)t$ and then passing through a low-pass filter.

(a) What is the output of the above demodulator?

(b) If $m(t) = \cos 2\pi f_m t$, sketch the spectrum of the output signal with the above demodulator. Assume $0 < \Delta f < f_m$. Carefully label the positions of impulses.

(a) $$m(t)\cos 2\pi f_c t \cdot 2\cos 2\pi(f_c+\Delta f)t$$

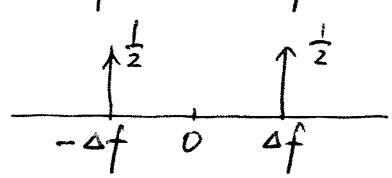
$$= m(t)\left[\cos 2\pi\Delta f t + \underbrace{\cos(4\pi f_c t + 2\pi\Delta f t)}_{\text{suppressed by low-pass filter}}\right]$$

Hence, the output is $m(t)\cos 2\pi\Delta f t$.

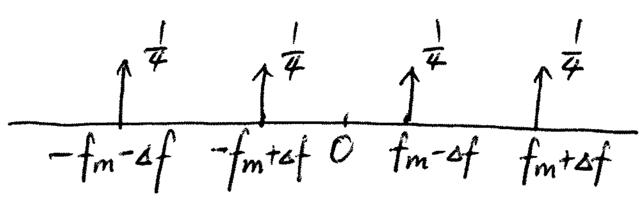
(b) If $m(t) = \cos 2\pi f_m t$, then

$$m(t)\cos 2\pi\Delta f t = \cos 2\pi f_m t \cos 2\pi\Delta f t$$

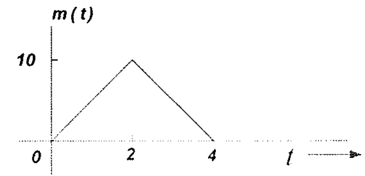
Spectrum of $\cos 2\pi\Delta f t$



spectrum of $\cos 2\pi\Delta f t \cos 2\pi f_m t$ is the above spectrum shifting to both sides by f_m.



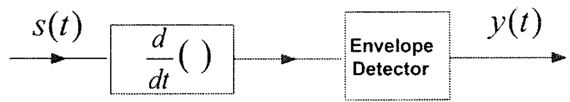
3. [10pt] Consider the PM modulated signal $s(t) = A_c \cos(2\pi f_c t + k_p m(t))$, where $k_p = \pi$ and $m(t)$ is given as follows.



(a) Find the frequency deviation of this PM signal $s(t)$.

(b) What is the bandwidth of this PM signal $s(t)$ in Hz. Assume the bandwidth of $m(t)$ is $B = 1/2$ Hz.

(c) If this signal goes through the following system, what would you get at the output of this system in time domain.



(a) $\Delta f = \frac{k_p \dot{m}_p}{2\pi}$

$(\dot{m}(t))_{max} = \frac{10}{2} = 5$

$(\dot{m}(t))_{min} = -\frac{10}{2} = -5$

Hence $\dot{m}_p = (\dot{m}(t))_{max} = 5$.

$\Delta f = \frac{k_p \dot{m}_p}{2\pi} = \frac{\pi \cdot 5}{2\pi} = \frac{5}{2}$ Hz.

(b) $B_{PM} = 2(\Delta f + B) = 2\left(\frac{5}{2} + \frac{1}{2}\right) = 6$ Hz.

(c) $\frac{d}{dt}(s(t)) = \frac{d}{dt}\left(A_c \cos(2\pi f_c t + k_p m(t))\right)$

$= -A_c (2\pi f_c + k_p \dot{m}(t)) \sin(2\pi f_c t + k_p m(t))$

Output of envelope detector is

$$A_c(2\pi f_c + k_p \dot{m}(t)) = \begin{cases} A_c(2\pi f_c + 5\pi) & 0 \le t \le 2 \\ A_c(2\pi f_c - 5\pi) & 2 \le t \le 4 \end{cases}$$

4. [20pt] An angle modulated signal has the form

$$s(t) = 100\cos[2\pi f_c t + 4\sin 2\pi f_m t]$$

where $f_c = 10$ MHz and $f_m = 1000$ Hz.

(a) If this is an FM signal, determine the frequency deviation and the signal bandwidth.

(b) Repeat part (a) if f_m is doubled.

(c) If this is a PM signal, determine the frequency deviation and the signal bandwidth.

(d) Repeat part (c) if f_m is doubled.

(e) Are your answers for (a) and (c) the same? Explain why or why not.

(a) FM: $k_f \int_0^t m(\alpha)\,d\alpha = 4\sin 2\pi f_m t$

Hence $k_f m(t) = 4 \cdot 2\pi f_m \cos 2\pi f_m t$ (taking derivative of the above equation)

$k_f m_p = 8\pi f_m$

$\Delta f = \dfrac{8\pi f_m}{2\pi} = 4$ kHz.

$B_{FM} = 2(\Delta f + B) = 2(4+1) = 10$ kHz.

Where $B = 1$ kz is obtained from $m(t) = \dfrac{8\pi f_m}{k_f} \cos 2\pi f_m t$.

(b) If f_m is doubled, both Δf and B are doubled.

Hence $B_{FM} = 2 \times 10$ kHz $= 20$ kHz.

(c) PM: $k_p m(t) = 4\sin 2\pi f_m t$.

$k_p \dot{m}(t) = 8\pi f_m \cos 2\pi f_m t$.

Hence $k_p \dot{m}_p = 8\pi f_m$. $\Delta f = \dfrac{k_p \dot{m}_p}{2\pi} = \dfrac{8\pi f_m}{2\pi} = 4$ kHz.

$B_{PM} = 2(\Delta f + B) = 10$ kHz.

(d) If f_m is doubled, both Δf and B are doubled.

Hence $B_{PM} = 20$ kHz.

(e) Answers for (a) and (c) 6are the same. This is because the given signal s(t) is in a general form of angle-modulated signal, and its bandwidth is the same no matter if it is viewed as FM or PM signal.

5. [15pt] The system shown in the figure is used to generate an AM signal. The message signal $m(t)$ has its maximum value at m_p (i.e., $m_p = \max|m(t)|$). Assume the bandwidth B of $m(t)$ satisfies $B << f_c$. The nonlinear device has an input-output characteristic

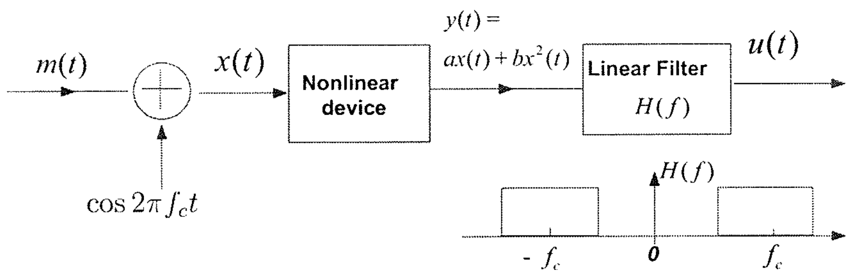
$$y(t) = ax(t) + bx^2(t).$$

The frequency response of the linear filter is also plotted in the figure.

(a) Express $y(t)$ in terms of $m(t)$ and the carrier $c(t) = \cos 2\pi f_c t$.

(b) Express $u(t)$ in terms of $m(t)$ and the carrier $c(t) = \cos 2\pi f_c t$.

(c) Is the output signal $u(t)$ an AM signal? If so, what is the modulation index?



(a)
$$\begin{aligned} y(t) &= a\,x(t) + b\,x^2(t) \\ &= a\left(m(t) + \cos 2\pi f_c t\right) + b\left(m(t) + \cos 2\pi f_c t\right)^2 \\ &= a\,m(t) + a\cos 2\pi f_c t + b\,m^2(t) + 2b\,m(t)\cos 2\pi f_c t + b\underbrace{\cos^2 2\pi f_c t}_{\frac{1+\cos 4\pi f_c t}{2}} \\ &= a\,m(t) + b\,m^2(t) + \frac{b}{2} + \left(a + 2b\,m(t)\right)\cos 2\pi f_c t + \frac{b}{2}\cos 4\pi f_c t \end{aligned}$$

(b) After the linear filter (bandpass filter), only spectrum around f_c is passed. Hence

$$u(t) = \left(a + 2b\,m(t)\right)\cos 2\pi f_c t$$

(c) Yes. The output is an AM signal.

$$\mu = \frac{2b\,m_p}{a}$$

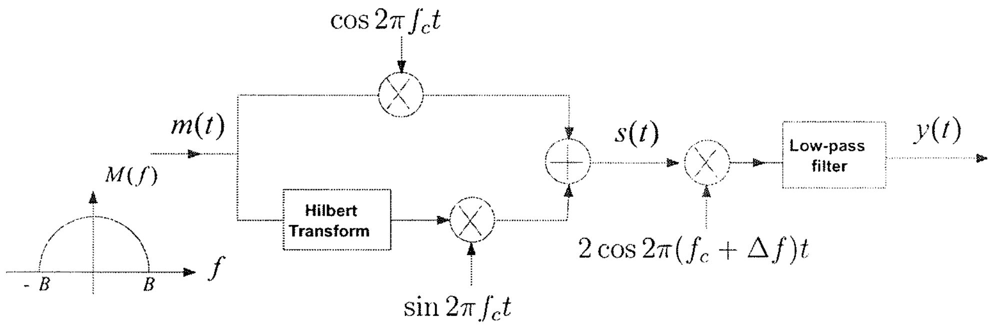
6. [15pt] Consider the following system.

(a) Write time domain expression of the signal $s(t)$.

(b) Draw the signal spectrum of $s(t)$, i.e., $S(f)$. (Hint: Which modulation scheme is used for generating $s(t)$?)

(c) Write the time domain expression of the signal $y(t)$.

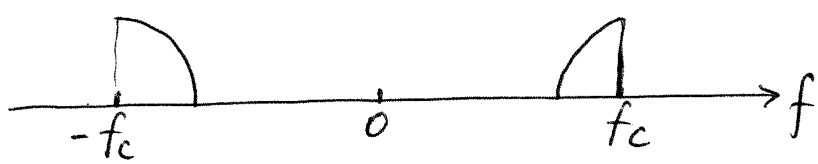
(d) Draw the signal spectrum of $y(t)$, i.e., $Y(f)$.



(a) $s(t) = m(t)\cos 2\pi f_c t + m_h(t)\sin 2\pi f_c t$

where $m_h(t)$ is Hilbert transform of $m(t)$

(b) $s(t)$ keeps the lower sideband of modulated $m(t)$.



(c) $2s(t)\cos 2\pi(f_c+\Delta f)t$

$= 2\left(m(t)\cos 2\pi f_c t + m_h(t)\sin 2\pi f_c t\right)\cos 2\pi(f_c+\Delta f)t$

$= m(t)\cos 2\pi\Delta f t + \underline{m(t)\cos 2\pi(2f_c+\Delta f)t}$

$+ m_h(t)(-\sin 2\pi\Delta f t) + \underline{m_h(t)\sin 2\pi(2f_c+\Delta f)t}$ } suppressed by LPF

Hence $y(t) = m(t)\cos 2\pi\Delta f t - m_h(t)\sin 2\pi\Delta f t$

(d) $y(t)$ is the upper sideband of modulated $m(t)$