State-Dependent Gaussian Interference Channels: Can State be Fully Cancelled? [1] [2]

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Abstract

The state-dependent Gaussian interference channel (IC) and Z-IC are investigated, in which two receivers are corrupted by the same but differently scaled states. The state sequence is noncausally known at both transmitters, but not known at either receiver. Three interference regimes are studied, i.e., the very strong, strong and weak regimes. In the very strong regime, the capacity region is characterized under certain channel parameters, by designing a cooperative dirty paper coding between the two transmitters to fully cancel the state. In the strong regime, points on the capacity region boundary are characterized under certain channel parameters by designing an achievable scheme based on rate splitting, layered dirty paper coding and successive state cancelation. In the weak regime, the sum capacity is obtained by independent dirty paper coding at two transmitters. For all the above regimes, the capacity achieves that of the IC/Z-IC without state. Comparison between the state-dependent regular IC and Z-IC suggests that even with one interference-free link, the Z-IC does not necessarily perform better, because dirty paper coded interference facilitates to cancel the state via cooperative dirty paper coding between the transmitters.

1 Introduction

State-dependent interference channels (ICs) with the noncausal state information at transmitters recently caught intensive attention. The main focus is on how to design encoding and decoding schemes to handle interference as well as canceling state at receivers in order to achieve as high transmission rates as possible. In particular, the Gel'fand-Pinsker binning scheme [1] and dirty paper coding [2] for state cancellation are typically jointly designed with interference cancelation schemes. Such design of achievable schemes has been well reflected in recent studies of state-dependent IC models. The IC model with the same state at two receivers has been studied in [3, 4]. Various achievable schemes have been designed and the

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corresponding achievable regions are compared in [3], and the gap between inner and outer bounds on the capacity region have been characterized within certain finite bits in [4]. The IC model with independent states at two receivers has been studied in [5], in which various achievable schemes have been studied, and the capacity region has been characterized for an asymptotic case when the power of the state sequence becomes large. Two state-dependent cognitive IC models have been studied in [6] and [7–9], respectively, in which inner and outer bounds on the capacity region for these models have been developed and have been shown to be tight for some special cases. A type of the state-dependent Z-IC was studied in [10, 11], in which only one receiver is corrupted by the state and the state information is known only to the mismatched transmitter. The bounds on the capacity region were derived, and the capacity region was characterized for certain cases. In [12], a state-dependent Z-interference broadcast channel was studied, in which one transmitter has one message for its corresponding receiver, and the other transmitter has two messages respectively for two receivers. Both receivers are corrupted by the same state, which is known to both transmitters. An achievable region was obtained based on lattice coding.

In this paper, we study the state-dependent Gaussian IC and Gaussian Z-IC with the two receivers corrupted by the same state, but scaled differently. The state sequence is noncausally known at both transmitters, but not known at either receiver. Differently from the previous studies of the same state-dependent IC model in [3,4] and a similar state-dependent Z-IC model in [12], our study focuses on the following two issues that were not addressed in the previous studies. (1) Under what channel parameters, the state at the two receivers can be fully canceled, and hence the capacity of the IC *without state* can be achieved by the corresponding *state-dependent* IC? Clearly, by understanding the above issue, the capacity region, or points on the boundary of the capacity region are characterized for the state-dependent IC. (2) Comparing to the state-dependent regular IC, does the state-dependent Z-IC have any advantage to cancel the state at the receivers due to an interference-free link to one receiver? Our main contributions in this paper lie in comprehensive investigation of the above two issues.

For the first issue, we note that the capacity region/the sum capacity has been characterized for the Gaussian IC *without* state in the following three regimes: very strong IC [13]; strong IC [14]; and a certain weak IC [15–17] (based on the technique developed in [18]); and for the Gaussian Z-IC in the corresponding regimes: very strong Z-IC [14, 19]; strong Z-IC [14, 19]; and weak Z-IC [19]. We study in this paper whether or not the capacity region/the sum capacity in these regimes are achievable when the two receivers' outputs are also corrupted by differently scaled state in additional to interference, and if so, what transmission schemes are capacity achieving. The challenge here lies in that the differently scaled state at two receivers requires novel design of dirty paper coding in order to fully cancel the *compound* state corruption at both receivers. Such difficulty has been observed in previous studies of differently scaled states at two receivers in [20, 21]. In this paper, we explore the properties of the IC in three regimes and characterize conditions on the channel parameters under which the capacity region/the sum capacity of the channel without state can be achieved, and hence the capacity region/the sum capacity for the channel with state is characterized.

More specifically, in the very strong interference regime, we characterize the conditions on the channel parameters, under which the capacity region of the IC and Z-IC channels *without* state can be achieved by the corresponding *state-dependent* ICs. The capacity of the state-dependent ICs are thus characterized under those cases. The novelty of the achievability lies in *cooperative dirty paper coding* between the two transmitters such that the interfered receiver decodes the dirty paper coded interference signal first to fully cancel the interference as well as to partially cancel the state, and then decodes the dirty paper encoded signal from its corresponding transmitter to cancel the remaining state. Here, cooperative dirty paper coding is possible because both transmitters know the state, which allows the cooperations. In the strong interference regime, we characterize the conditions on the channel parameters, under which points on the capacity region boundary of the channel *without* state can be achieved. Hence, these points also lie on the capacity region boundary of the state-dependent channel. The main difference of the strong regime from the very strong regime is the additional sum rate constraint in the capacity region, and cooperative dirty paper coding that we design for the very strong regime does not fully cancel influence of the state on the sum rate bound. The novelty of our scheme here lies in exploiting the fact that the sum rate boundary is due to the multiple-access decoding requirement at receiver 1 and can be achieved by rate splitting. Thus, we design *layered dirty paper coding* and *successive state cancelation* in order to fully cancel the state at receiver 1, with the expectation that such a coding scheme does not introduce extra bounds for receiver 2 to decode the messages. For the weak interference regime, we obtain the sum capacity. The result is based on the observation that treating interference as noise is optimal for the IC without state, and hence independent dirty paper coding at two transmitters to cancel the state at their corresponding receivers (treating the interference as noise) can achieve the same sum capacity as the IC without state.

For the second issue mentioned above, comparing the state-dependent regular IC with Z-IC, our results provide a few interesting insights. In the very strong interference regime, our comparison suggests that it is easier to fully cancel the state for the regular IC than the Z-IC, which may appear counter intuitive. In fact, it is reasonable because one more interference link in the regular IC helps the interfered receiver to partially cancel the state via its decoding of the dirty paper coded interference. This implies that interference here is useful and can be exploited for cooperative state cancelation. In the strong interference regime, our comparison suggests that the Z-IC does not have advantage to cancel the state more easily than the regular IC. This is because the scheme that achieves points on the capacity region boundary is designed to cancel the state for the interfered receiver for both channels, and then the advantage of the Z-IC at the other receiver is not significant due to the state interference that is not fully cancelled. In the weak interference regime, the sum capacity is characterized for the state-dependent Z-IC over the entire regime, whereas it is characterized for the regular IC only under certain conditions. This is due to the more understanding in the state-of-the-art for the Z-IC without state.

The rest of the paper is organized as follows. In Section 2, we describe the channel model and explain the notation used in this paper. In Sections 3, 4, and 5, we respectively present our results for the very strong, strong, and weak Gaussian state-dependent IC and Z-IC. Finally, in Section 6, we conclude with a few remarks.

2 Channel Model

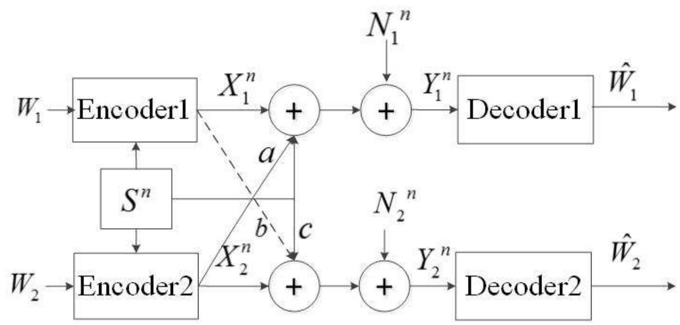

Figure 1: The state-dependent IC (with the dashed line) and Z-IC (without the dashed line)

We study the state-dependent regular IC (as shown in Fig. 1 with the dashed line) and the state-dependent Z-IC (without the dashed line). For both channels, transmitter 1 sends a message W_1 to receiver 1, and transmitter 2 sends a message W_2 to receiver 2. The channel is corrupted by an independent and identically distributed (i.i.d.) state sequence S^n, which is assumed to be known *noncausally* at both transmitters. Transmitter 1 maps a message $w_1 \in \{1, \ldots, 2^{nR_1}\}$ and a state sequence s^n to an input x_1^n, and transmitter 2 maps a message $w_2 \in \{1, \ldots, 2^{nR_2}\}$ and a state sequence s^n to an input x_2^n. These two inputs are then sent over the memoryless IC characterized by $P_{Y_1Y_2|X_1X_2S}$. For the Z-IC, only receiver 1 is interfered by transmitter 2's signal, while receiver 2 is free from interference. Hence, the channel is characterized by $P_{Y_1|X_1X_2S}$ and $P_{Y_2|X_2S}$. For both channels, receiver 1 is required to decode W_1 and receiver 2 is required to decode W_2, with the probability of error approaching zero as the codeword length n goes to infinity. The *capacity region* is defined to be the closure of the set of all achievable rate pairs (R_1, R_2).

In this paper, we study the Gaussian channel with the outputs at receivers 1 and 2 for one channel use given by

$$\begin{aligned} Y_1 &= X_1 + aX_2 + S + N_1 \\ Y_2 &= bX_1 + X_2 + cS + N_2 \end{aligned} \tag{1}$$

where a, b and c are constants, the noise variables N_1 and N_2 are Gaussian distributed with zero mean and unit variance, i.e., $N_1,\ N_2 \sim \mathcal{N}(0, 1)$, and the state variable S is Gaussian distributed with mean zero and variance Q, i.e., $S \sim \mathcal{N}(0, Q)$. Both the noise and state variables are i.i.d. over channel uses. The channel inputs X_1 and X_2 are subject to the average power constraints P_1 and P_2, i.e., $\frac{1}{n}\sum_{i=1}^n X_{1i}^2 \leqslant P_1$, and $\frac{1}{n}\sum_{i=1}^n X_{2i}^2 \leqslant P_2$. For the Z-IC, the channel parameter $b = 0$, and thus receiver 2 is not interfered by transmitter 1.

The goal of this paper is to identify conditions on the channel parameters (a, b, c, P_1, P_2, Q) under which the capacity region, points on the boundary of the capacity region, or the sum capacity can be characterized for the state-dependent Gaussian IC and Z-IC.

3 Very Strong Interference Regime

In this section, we study the state-dependent regular IC and Z-IC in the very strong regime, and characterize the conditions under which the capacity region can be obtained, i.e., the capacity region of the IC *without* state can be achieved. We also compare the results of the two channels.

3.1 State-Dependent Regular IC

In this subsection, we study the state-dependent regular IC in the very strong regime, in which the channel parameters satisfy

$$\begin{aligned} &P_1 + a^2 P_2 + 1 \geq (1+P_1)(1+P_2) \\ &b^2 P_1 + P_2 + 1 \geq (1+P_1)(1+P_2). \end{aligned} \tag{2}$$

In such a regime, the channel *without state* is the very strong IC, and its capacity region has been characterized in [13], which contains rate pairs (R_1, R_2) satisfying

$$R_1 \leqslant \frac{1}{2}\log(1+P_1) \tag{3a}$$

$$R_2 \leqslant \frac{1}{2}\log(1+P_2). \tag{3b}$$

In this case, the two users achieve the point-to-point channel capacity even with interference. Our focus here is to study under what conditions on the channel parameters we can design schemes for the *state-dependent* IC to achieve the above capacity region, i.e., the state at receivers can be fully cancelled. Clearly, in this case, the above capacity region also serves as the capacity region for the state-dependent channel.

There are two challenges here. (1) Since the state are scaled differently at two receivers, each transmitter needs to deal with the compound state corruption in two receivers. (2) The scheme to achieve the capacity region for the very strong IC without state suggests that the receivers decode the interference first, and then cancel it from the received output so that decoding of the intended input does not experience interference. For the state-dependent channel, if both transmitters employ dirty paper coding, receivers decode only auxiliary random variables, but not the exact input of the other transmitter. Hence, canceling the signal interference would cause certain left-over state interference.

The novelty of our scheme to achieve the point-to-point channel capacity for each user lies in designing *cooperative* dirty paper coding between the two transmitters such that (1) the two transmitters cooperatively cancel the compound states at the two receivers, and furthermore (2) each transmitter design its dirty paper input based on the original state plus the left-over interference by decoding the other transmitter's dirty paper coded interference. The cooperation between the transmitters is possible due to the state information known to both transmitters.

In the following, we first design an achievable scheme for the discrete memoryless channel, which is useful for the Gaussian channel. The two transmitters encode their messages W_1 and W_2 into two auxiliary random variables U and V, respectively, based on Gel'fand-Pinsker binning scheme [1]. Since the channel satisfies the very strong interference condition, each receiver first decodes the auxiliary random variable corresponding to the message intended for the other receiver, and then decodes its own message by decoding the auxiliary random variable for itself. For instance, receiver 1 first decodes V, then uses it to cancel the message interference and state interference, and finally decodes its message by decoding U. Such an achievable scheme yields the following achievable region.

Proposition 1. *For the state-dependent IC with state noncausally known at both transmitters, the achievable region consists of rate pairs* (R_1, R_2) *satisfying:*

$$R_1 \leqslant \min\{I(U;Y_1V), I(U;Y_2)\} - I(U;S) \tag{4a}$$
$$R_2 \leqslant \min\{I(V;Y_2U), I(V;Y_1)\} - I(V;S) \tag{4b}$$

for some distribution $P_{SUVX_1X_2Y_1Y_2} = P_S P_{U|S} P_{X_1|US} P_{V|S} P_{X_2|VS} P_{Y_1Y_2|X_1X_2S}$*, where* U *and* V *are auxiliary random variables.*

Proof. See Appendix A. □

By choosing joint Gaussian distributions for the auxiliary random variables and the channel inputs in the achievable region given in Proposition 1, we can obtain an achievable region for the Gaussian channel. In particular, U is designed to deal with the state interference for Y_1 after cancelling V, and V is designed to deal with the state interference for Y_2 after cancelling U. Therefore, coefficients in dirty paper coding of U and V are jointly designed to cancel the states at the two receivers. Furthermore, by requiring $I(U;Y_1V) \geq I(U;Y_1)$ and $I(V;Y_1U) \geq I(V;Y_1)$ in (4a) and (4b), the resulting region is the same as the capacity region of the channel without state, and thus the capacity region of the state-dependent IC is established. We state this result in the following theorem.

Theorem 1. *Consider the state-dependent Gaussian IC with state noncausally known at both transmitters. If the channel parameters* (a, b, c, P_1, P_2, Q) *satisfy the following conditions:*

$$\frac{(b^2P_1 + P_2 + c^2Q + 1)}{(1+P_2)(1 + \frac{(1+P_2)(c+cP_1-bP_1)^2Q + QP_1(1+P_2-acP_2)^2}{((1+P_1)(1+P_2)-abP_1P_2)^2})} \geqslant 1 + P_1 \tag{5a}$$
$$\frac{(P_1 + a^2P_2 + Q + 1)}{(1+P_1)(1 + \frac{P_2(c+cP_1-bP_1)^2Q + Q(1+P_1)(1+P_2-acP_2)^2}{((1+P_1)(1+P_2)-abP_1P_2)^2})} \geqslant 1 + P_2, \tag{5b}$$

then the capacity region consists of rate pairs (R_1, R_2) *satisfying* (3a) *and* (3b)*, i.e., is the same as the point-to-point capacity for both receivers.*

Proof. In Proposition 1, we set U and V as $U = X_1 + \alpha S$, $V = X_2 + \beta S$, where X_1, X_2 and S are independent Gaussian variables with mean zero and variances P_1, P_2 and S, respectively. We then design α based on dirty paper coding for $Y_1' = Y_1 - aV = X_1 + (1 - a\beta)S + N_1$, and design β based on dirty paper coding for $Y_1' = Y_1 - bU = X_2 + (c - b\alpha)S + N_2$. We further require α and β to satisfy the following conditions:

$$\frac{\alpha}{1 - a\beta} = \frac{P_1}{P_1 + 1} \tag{6a}$$

$$\frac{\beta}{c - b\alpha} = \frac{P_2}{P_2 + 1}. \tag{6b}$$

By solving the equations (6a) and (6b), we have

$$\alpha = \frac{P_1(1 + P_2 - acP_2)}{(1 + P_1)(1 + P_2) - abP_1P_2}$$

$$\beta = \frac{cP_2(C(1 + P_1) - bP_1)}{(1 + P_1)(1 + P_2) - abP_1P_2}.$$

Then the bounds in equations (4a) and (4b) becomes

$$R_1 \leqslant \frac{1}{2}\log(1 + P_1)$$

$$R_2 \leqslant \frac{1}{2}\log(1 + P_2), \tag{7}$$

if

$$\frac{1}{2}\log(1 + P_1) \leqslant I(U; Y_2) - I(U; S)$$

$$\frac{1}{2}\log(1 + P_2) \leqslant I(V; Y_1) - I(V; S). \tag{8}$$

By computing the mutual information terms in the above equations based on the chosen distributions for U and V, we obtain the conditions given in the theorem. Such an achievable region is therefore the capacity region, because it is the same as the corresponding channel without state. This can be formally shown by following steps similar to those in Appendix G. □

We note that (5a) and (5b) can be viewed as the conditions that define the very strong regime for the state-dependent Gaussian IC. If $Q = 0$, i.e., the state power is zero, these conditions reduce to those given in (2) that define the very strong regime for the Gaussian IC without state. Although the conditions (5a) and (5b) are expressed in complicated forms, they can be easily checked numerically. We provide numerical illustration in Section 3.3. We further provide the following result for the state-dependent symmetric Gaussian IC as a special case, for which the very strong condition becomes much simpler and more intuitive, i.e., the interference should be above a certain threshold.

Corollary 1. *For the state-dependent symmetric Gaussian IC with state noncausally known at the transmitters, i.e., $a = b$, $c = 1$, and $P_1 = P_2$, the capacity region contains rate pairs (R_1, R_2) satisfying*

$$R_1 \leqslant \frac{1}{2}\log(1 + P)$$

$$R_2 \leqslant \frac{1}{2}\log(1 + P), \tag{9}$$

if $a \geq a_{th}$*, where* a_{th} *solves the following equation*

$$\frac{(P+a^2P+Q+1)(1+P+aP)^2}{(1+P)[(1+P+aP)^2+Q(1+2P)]} = 1+P. \tag{10}$$

Proof. If $a = b$ and $c = 1$, then the conditions (5b) and (5a) reduce to the following single condition:

$$\frac{(P+a^2P+Q+1)(1+P+aP)^2}{(1+P)[(1+P+aP)^2+Q(1+2P)]} \geqslant 1+P. \tag{11}$$

Such a condition is equivalent to the one given in the corollary. □

3.2 State-Dependent Z-IC

In this subsection, we study the state-dependent Z-IC, i.e., $b = 0$, in the very strong regime, in which the channel parameter satisfies

$$a^2 > 1 + P_1. \tag{12}$$

Under the above condition, the channel *without* state is very strong, and its capacity region contains rate pairs (R_1, R_2) satisfying (3a) and (3b), i.e., the two users achieve the point-to-point channel capacity.

Similarly to the regular IC, we study under what conditions the state-dependent Z-IC achieves the capacity region of the channel without state, i.e., the state at the receivers can be fully cancelled. We also design cooperative dirty paper coding between the two transmitters, which encodes the messages W_1 and W_2 into two auxiliary random variables U and V, respectively. The difference from the scheme for the regular IC lies in the fact that since receiver 2 is interference free, V can be designed to fully cancel the state at receiver 2. Then receiver 1 first decodes the auxiliary random variable V to cancel the interference as well as partial state, and then decodes its own message and cancels the remaining state by decoding the auxiliary random variable U. Based on this achievable scheme, we have the following achievable region for the discrete memoryless channel.

Proposition 2. *For the state-dependent Z-IC with state noncausally known at both transmitters, the achievable region consists of rate pairs* (R_1, R_2) *satisfying:*

$$\begin{aligned} &R_1 \leqslant I(U;VY_1) - I(U;S), \\ &R_2 \leqslant I(V;Y_2) - I(V;S) \end{aligned} \tag{13}$$

for some distribution $P_S P_{U|S} P_{V|S} P_{X_1|US} P_{X_2|VS} P_{Y_1|X_1X_2S} P_{Y_2|X_2S}$ *that satisfies* $I(V;Y_2) \leqslant I(V;Y_1)$.

Proof. See Appendix B. □

By choosing the joint Gaussian distribution for the auxiliary random variables and the channel inputs in the achievable region in Proposition 2, we obtain the achievable region for the state-dependent Gaussian Z-IC. In particular, the auxiliary random variable V is designed to deal with the state interference for Y_2, but U is designed to deal with the state interference for Y_1 after cancelling V. By further comparing this achievable region with the capacity region of the Z-IC *without* state, we obtain the following capacity result.

Theorem 2. *For the state-dependent Gaussian Z-IC with state noncausally known at both transmitters, if its channel parameters* (a, c, P_1, P_2, Q) *satisfy the following conditions:*

$$\frac{P_2(a^2P_2 + P_1 + 1)}{P_2Q(1-\alpha)^2 + (P_2 + \alpha^2Q)(P_1 + 1)} \geqslant 1 + P_2,$$

where $\alpha = \frac{P_2}{P_2+1}c$*, then the capacity region consists of rate pairs* (R_1, R_2) *satisfying* (3a) *and* (3b)*.*

Proof. We set U and V in Proposition 2 as $U = X_1 + \beta S$, $V = X_2 + \alpha S$, where X_1, X_2 and S are independent Gaussian variables with mean zero and variances P_1, P_2 and Q, respectively, and set α and β to be

$$\alpha = \frac{P_2}{(1+P_2)}b, \quad \beta = \frac{P_1}{1+P_1}(1-\alpha).$$

Substituting the above choice of the Gaussian distribution into Proposition 2 yields the desired region and the condition in Theorem 2.

Since such an achievable region is the same as the capacity region of the corresponding channel without state, it can be shown to be the capacity region of the state-dependent channel. □

3.3 Comparison of State-Dependent Regular IC and Z-IC

In this subsection, we compare the result in Theorem 1 for the state-dependent *regular* IC and the result in Theorem 2 for the state-dependent *Z-IC*.

We set $P_1 = 1$, $P_2 = 1$, and $Q = 1.2$ for both channels, and set the additional interference link in the regular IC to have the channel gain $b = 4$ such that it does not affect a fair comparison. In Fig. 2, we plot the range of parameter pairs (a, c) under which the two point-to-point channel capacities can be achieved for both state-dependent *regular IC* and *Z-IC*. The ranges between the two solid lines and between the two dashed lines respectively correspond to the regular IC and Z-IC. It is clear that the regular IC has a larger range than the Z-IC particularly for large a. Such observation suggests that it is easier to fully cancel the state for the regular IC than the Z-IC, which may appear counter intuitive, since the state-dependent Z-IC possesses an interference free link. In fact, it is reasonable, because receiver 2 in the regular IC can decode the dirty paper coded signal of transmitter 1 due to the very strong interference, via which it can cancel certain amount of state. In this way, the one more interference link to receiver 2 in the regular IC helps receiver 2 to cancel the state.

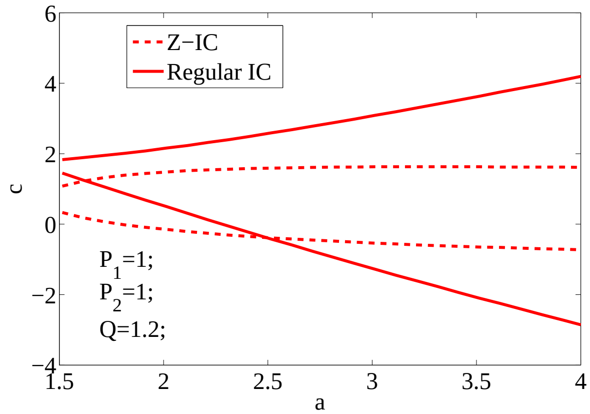

Figure 2: Conditions on channel parameters (a, c) under which the state-dependent Gaussian regular IC and Z-IC achieve the capacity of the corresponding channel without state in the very strong regime.

4 Strong Interference Regime

Since the very strong IC is studied separately in Section 3, in this section, we study the state-dependent regular IC and Z-IC in the strong, but not very strong regime, and characterize the conditions under which points on the capacity region boundary can be obtained. We then compare the results for the regular IC and Z-IC.

4.1 State-Dependent Regular IC

In this subsection, we study the state-dependent regular IC in the strong but not very strong regime, in which the channel parameters satisfy

$$\begin{aligned} &a \geq 1, \quad b \geq 1, \\ &\min\{P_1 + a^2P_2 + 1, b^2P_1 + P_2 + 1\} \leqslant (1 + P_1)(1 + P_2). \end{aligned} \tag{14}$$

Without loss of generality, we assume that $P_1 + a^2P_2 + 1 \leqslant b^2P_1 + P_2 + 1$. Under the above conditions, the IC *without* state is strong, and the capacity region was characterized in [14], which contains rate pairs (R_1, R_2) satisfying

$$\begin{aligned} &R_1 \leqslant \frac{1}{2}\log(1 + P_1), \qquad R_2 \leqslant \frac{1}{2}\log(1 + P_2), \\ &R_1 + R_2 \leqslant \frac{1}{2}\log(1 + P_1 + a^2P_2). \end{aligned} \tag{15}$$

The above capacity is achieved by requiring both receivers to decode both messages, and hence the capacity region is the intersection of the capacity regions of two multiple-access channels. We illustrate such a capacity region in Fig. 3, as the pentagon O-A-B-E-F-O.

Our goal here is to study whether points on the boundary of such a pentagon (i.e., the capacity region boundary of the IC *without* state) can be achieved by the corresponding

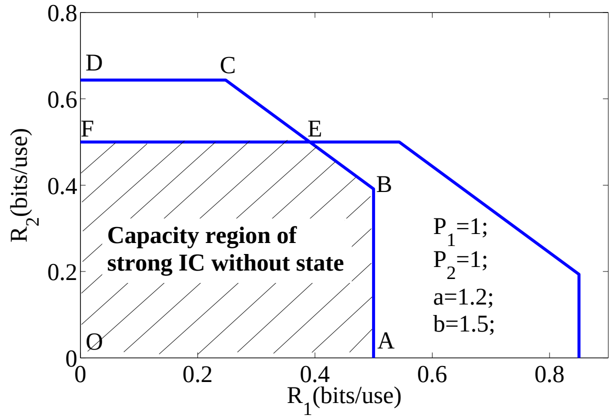

Figure 3: Capacity region of the strong IC without state

state-dependent IC. The main difference of the strong regime from the very strong regime studied in Section 3 is the additional sum rate constraint in the capacity region. Although the cooperative dirty paper coding scheme that we design for the very strong regime fully cancels the state in the single-user rate bounds, it does not fully cancel the state in the sum rate bound. Thus, new schemes need to be designed here in order for the state-dependent IC to achieve the sum rate boundary of the capacity region of the IC without state, i.e., the line B-E in Fig. 3. Then the points on the line A-B and the line F-E are achievable if the two corner points B and E on the sum rate boundary are achievable.

The idea of our achievable scheme is to exploit the fact that the sum rate boundary B-E is due to the decoding requirement at receiver 1 (as a receiver of the multiple access channel), and hence every point on B-E can be achieved by message splitting and successive cancelation. Thus, for the state-dependent channel, in addition to rate splitting, the novel ingredients of our achievable scheme lie in *layered dirty paper coding* and *successive state cancelation* aiming at fully canceling the state at receiver 1. If such a coding scheme does not introduce extra bounds for receiver 2 to decode the two messages, then the sum rate boundary can be achieved.

Based on the above idea, we first design an achievable scheme for the corresponding discrete memoryless channel which is useful for studying the Gaussian channel. We split the message W_1 into two parts W_{11} and W_{12}, which are encoded into the auxiliary random variables U_1 and U_2 successively as in Proposition 3 using Gelfand-Pinsker binning. We also split the message W_2 into two parts W_{21} and W_{22}, which are encoded into the auxiliary random variables V_1 and V_2 successively as in Proposition 3 using Gelfand-Pinsker binning scheme. Both receivers decode both messages with reasonable decoding orders, such that the decoding capability of the two receivers are accommodated. As an illustration, we next adopt the decoding order $W_{11}, W_{21}, W_{22}, W_{12}$ at receiver 1 and the decoding order $W_{21}, W_{11}, W_{12}, W_{22}$ at receiver 2. The resulting achievable rate region is given in the following Proposition.

Proposition 3. *For the state-dependent IC with state noncausally known at both transmit-*

ters, an achievable region consists of rate pairs (R_1, R_2) *satisfying:*

$$\begin{aligned}R_1 \leqslant\ & \min\{I(U_1;Y_1), I(U_1;V_1Y_2)\}\\ &+\min\{I(U_2;V_1V_2Y_1|U_1), I(U_2;V_1Y_2|U_1)\} - I(U_1U_2;S)\\ R_2 \leqslant\ & \min\{I(V_1;Y_2), I(V_1;U_1Y_1)\}\\ &+\min\{I(V_2;U_1U_2Y_2|V_1), I(V_2;U_1Y_1|V_1)\} - I(V_1V_2;S)\end{aligned} \tag{16}$$

for some distribution

$$P_{SU_1U_2V_1V_2X_1X_2Y_1Y_2} = P_S P_{U_1|S} P_{U_2|SU_1} P_{X_1|U_1U_2S} P_{V_1|S} P_{V_2|SV_1} P_{X_2|V_1V_2S} P_{Y_1Y_2|SX_1X_2}$$

where U_1*,* U_2*,* V_1*, and* V_2 *are auxiliary random variables.*

Proof. See Appendix C. □

Remark 1. *A more comprehensive achievable region can be obtained by taking the convex hull of the union over achievable regions resulting from all possible decoding orders of messages at the two receivers.*

Proposition 3 provides an example achievable region, based on which we next show that the designed scheme achieves the capacity region or partial boundary of the capacity region for the state-dependent Gaussian IC under certain conditions on channel parameters. Namely, we characterize the conditions on the channel parameters under which points on the sum rate boundary of the IC without state (i.e., the line B-E in Fig. 3) can be achieved by the state-dependent Gaussian IC.

We note that any rate point on the line B-E can be characterized by

$$\begin{aligned}R_1 &= \frac{1}{2}\log\left(1+\frac{P_1'}{a^2P_2''+P_1''+1}\right)+\frac{1}{2}\log\left(1+P_1''\right)\\ R_2 &= \frac{1}{2}\log\left(1+\frac{a^2P_2'}{a^2P_2''+P_1+1}\right)+\frac{1}{2}\log\left(1+\frac{a^2P_2''}{P_1''+1}\right)\end{aligned} \tag{17}$$

for some $P_1', P_1'', P_2', P_2'' \geqslant 0$, $P_1'+P_1'' \leqslant P_1$, and $P_2'+P_2'' \leqslant P_2$.

In order to achieve any rate point given in (17), we design layered dirty paper coding for the auxiliary random variables U_1, V_1, V_2, and U_2 in order to successively decode messages and cancel the state at receiver 1. More specifically, dirty paper coding for U_1 is designed to cancel the state treating all other variables as noise, and then V_1, V_2 and U_2 are designed to successively cancel the residual state after subtracting the previously decoded auxiliary random variables from Y_1. Furthermore, by requiring the rate bounds due to decoding at receiver 2 to be larger than those due to decoding at receiver 1, the rate point of interest is thus achievable for the state-dependent IC. We state this result in the following theorem.

Theorem 3. *Any rate point given in* (17) *with the parameters* $(P_1', P_1'', P_2', P_2'')$ *is on the capacity region boundary of the state-dependent IC if the channel parameters satisfy the* *following conditions*

$$\frac{1}{2}\log\left(1+\frac{P_1'}{P_1''+a^2P_2+1}\right) \leqslant I(U_1;V_1Y_2) \tag{18a}$$

$$\frac{1}{2}\log(1+P_1'') \leqslant I(U_2;V_1Y_2|U_1) \tag{18b}$$

$$\frac{1}{2}\log\left(1+\frac{a^2P_2'}{P_1''+a^2P_2''+1}\right) \leqslant I(V_1;Y_2) \tag{18c}$$

$$\frac{1}{2}\log\left(1+\frac{a^2P_2''}{P_1''+1}\right) \leqslant I(V_2;U_1U_2Y_2|V_1) \tag{18d}$$

where the mutual information terms in the above conditions are computed based on the following auxiliary random variables

$$\begin{aligned} U_1 &= X_1' + \alpha_1 S, \quad U_2 = X_1'' + \alpha_2 S \\ V_1 &= aX_2' + \beta_1 S, \quad V_2 = aX_2'' + \beta_2 S \end{aligned} \tag{19}$$

where X_1', X_1'', X_2', X_2'' *are independent Gaussian variables with mean zero and variances* P_1', P_1'', P_2' *and* P_2''*, correspondingly,* $X_1 = X_1' + X_1''$*,* $X_2 = X_2' + X_2''$*, and* $\alpha_1, \alpha_2, \beta_1$ *and* β_2 *are given by*

$$\begin{aligned} \alpha_1 &= \frac{P_1'}{P_1+a^2P_2+1}, \quad \alpha_2 = \frac{P_1''}{P_1+a^2P_2+1} \\ \beta_1 &= \frac{a^2P_2'}{P_1+a^2P_2+1}, \quad \beta_2 = \frac{a^2P_2''}{P_1+a^2P_2+1}. \end{aligned}$$

Proof. The achievability follows from Proposition 3 by choosing the auxiliary random variables U_1, U_2, V_1, and V_2 as in (19) based on the successive dirty paper coding for removing the state from the received signal Y_1 so that the rate point given in (17) is achievable at receiver 1. For this rate point to be achievable also at receiver 2, following Proposition 3, the following conditions should be satisfied

$$I(U_1;Y_1) \leqslant I(U_1;V_1Y_2) \tag{20a}$$

$$I(U_2;V_1V_2Y_1|U_1) \leqslant I(U_2;V_1Y_2|U_1) \tag{20b}$$

$$I(V_1;U_1Y_1) \leqslant I(V_1;Y_2) \tag{20c}$$

$$I(V_2;U_1Y_1|V_1) \leqslant I(V_2;U_1U_2Y_2|V_1). \tag{20d}$$

By substituting the auxiliary random variables defined in (19) into (20a)-(20d), we obtain the conditions (18a)-(18d) on the channel parameters, under which the given boundary point is achievable by the state-dependent IC. Thus, such a point is on the capacity region boundary, because it is on the capacity boundary of the channel without state, which serves as an outer bound. Formal justification can follow steps similar to those in Appendix G. □

We note that the mutual information terms in Theorem 4 can be explicitly computed in close forms. Thus, Theorem 4 provides a computable way for checking whether any point on the sum rate boundary of the capacity of the IC without state is also on the capacity boundary for the corresponding state-dependent channel under certain channel parameters. We provide an example range of parameters in Section 4.3.

4.2 State-Dependent Z-IC

In this subsection, we study the state-dependent Z-IC (i.e., $b = 0$) in the strong but not very strong regime, in which the channel parameters satisfy

$$1 \leqslant a^2 \leqslant (1 + P_1). \tag{21}$$

Under the above conditions, the Z-IC *without* state is strong (but not very strong Z-IC), and the capacity region is characterized in [14], which contains rate pairs (R_1, R_2) satisfying

$$\begin{aligned} & R_1 + R_2 \leqslant \frac{1}{2}\log(1 + P_1 + a^2 P_2) \\ & R_1 \leqslant \frac{1}{2}\log(1 + P_1), \quad R_2 \leqslant \frac{1}{2}\log(1 + P_2). \end{aligned} \tag{22}$$

The above capacity region is illustrated in Fig. 4 as the pentagon O-A-B-E-F-O, which is obtained by requiring receiver 1 to decode both messages and receiver 2 to decode the message W_2.

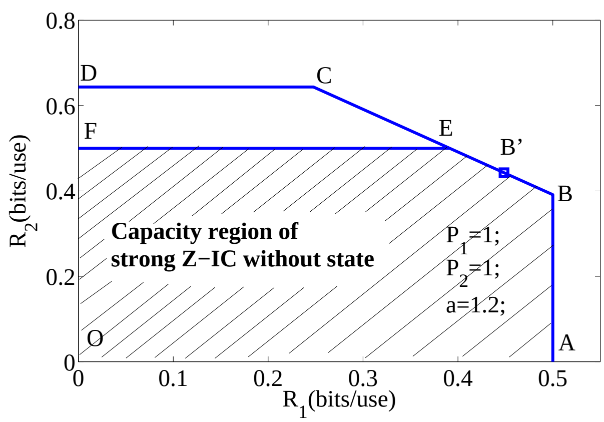

Figure 4: Capacity region of the strong Z-IC without state

Similarly to the regular IC, our goal here is also to study whether the points on the boundary of such a pentagon (i.e., the capacity region boundary of the Z-IC *without* state) can be achieved by the corresponding *state-dependent* Z-IC. We focus on the sum rate boundary of the pentagon (i.e., the line B-E in Fig. 4), and then the points on the line A-B and the line E-F are achievable if the two corner points B and E are achievable. We first design an achievable scheme for the state-dependent discrete memoryless Z-IC following the same idea as that for the regular IC based on rate splitting, layered dirty paper coding and successive state cancelation aiming at fully canceling the state at receiver 1. The only difference lies in that receiver 2 here decodes only W_{21} and W_{22}. Such a scheme then yields the following achievable rate region.

Proposition 4. *For the state-dependent Z-IC with state noncausally known at both transmitters, an achievable region consists of rate pairs* (R_1, R_2) *satisfying:*

$$\begin{aligned} R_1 &\leqslant I(U_1; V_1Y_1) + I(U_2; V_1V_2Y_1|U_1) - I(U_1, U_2; S) \\ R_2 &\leqslant \min\{I(V_1; Y_2), I(V_1; Y_1)\} \\ &\quad + \min\{I(V_2; Y_2|V_1), I(V_2; U_1Y_1|V_1)\} - I(V_1V_2; S) \end{aligned} \tag{23}$$

for some distribution $P_{SU_1U_2V_1V_2X_2X_1Y_2Y_1} = P_S P_{U_1U_2|S} P_{V_1V_2|S} P_{X_1|U_1U_2S} P_{X_2|V_1V_2S} P_{Y_1|SX_1X_2} P_{Y_2|SX_2}$, *where* U_1, U_2, V_1 *and* V_2 *are auxiliary random variables.*

Proof. See Appendix D. □

Now specializing Proposition 4 to the Gaussian case yields an achievable region, based on which we can check if and under what conditions the points on the line B-E in Fig. 4 are achievable. Since points on the line B-E can also be characterized in (17) , we thus follow the same design of layered dirty paper coding for the auxiliary random variables U_1, V_1, V_2, and U_2 as that for the regular IC in order to fully cancel the state at receiver 1 successively. Then by requiring the decoding bounds at receiver 2 to be larger than those of receiver 1, points on B-E can be shown to be achievable by the state-dependent Z-IC. We state this result in the following theorem.

Theorem 4. *Any rate point characterized in* (17) *with the parameters* $(P_1', P_1'', P_2', P_2'')$ *is on the capacity region boundary of the state-dependent Gaussian Z-IC with state noncausally known at the transmitters if the channel parameters satisfy the following conditions*

$$1 + \frac{a^2P_2'}{a^2P_2'' + P_1 + 1} \leqslant \frac{a^2P_2'(P_2 + b^2Q + 1)}{P_2'(ab-\alpha)^2 + (P_2''+1)(a^2P_2' + \alpha^2 Q)} \tag{24a}$$

$$1 + \frac{a^2P_2''}{P_1''+1} \leqslant \frac{a^2P_2''[P_2'(a^2P_2'' + (ab-\alpha)^2Q + a^2) + \alpha^2Q(P_2''+1)]}{a^2P_2''(\alpha^2Q + a^2P_2') + (a^2b - a\alpha - a\gamma)^2P_2'P_2''Q + a^2\gamma^2P_2'Q} \tag{24b}$$

where $\alpha = \frac{a^2P_2'}{a^2P_2+P_1+1}$, *and* $\gamma = \frac{a^2P_2''}{a^2P_2+P_1+1}$.

Proof. In order to achieve a rate point given in (17) with the parameters $(P_1', P_1'', P_2', P_2'')$, we apply Proposition 4 and choose the auxiliary random variables U_1, U_2, V_1, and V_2 based on the dirty paper coding as in (19) so that the state in the received signal Y_1 can be fully canceled.

In order for receiver 2 to decode at this rate point (without introducing more constraints on the rates), due to Proposition 4, the following conditions should be satisfied

$$I(V_1; Y_1) \leqslant I(V_1; Y_2), \quad I(V_2; U_1Y_1|V_1) \leqslant I(V_2; Y_2|V_1). \tag{25}$$

By substituting the auxiliary random variables defined in (19) into (25), the conditions (24a) and (24b) on the channel parameters can be obtained, under which the rate point of interest is achievable over the state-dependent Z-IC. Thus, such a rate point is on the capacity region boundary, because it is on the capacity region boundary of the channel without state, which serves as an outer bound. □

Theorem 4 provides the conditions on the channel parameters under which a certain given point is on the capacity region boundary. It is also of interest to characterize the rate points that are on the capacity region boundary for a given set of channel parameters. The following proposition provides a useful property that further yields characterization of a line segment on the capacity region boundary for a given set of channel parameters in Corollary 2.

Proposition 5. *For the state-dependent Gaussian Z-IC with state noncausally known at both transmitters, if a point (say B') on the line $B-E$ in Fig. 4 satisfies the conditions in Theorem 4, i.e., it is on the capacity region boundary, then the point B is also on the capacity region boundary, and thus the line segment $B'-B$ is on the capacity region boundary.*

Proof. See Appendix E. □

Based on Proposition 5, we characterize a segment on the capacity region boundary in the following corollary.

Corollary 2. *For the state-dependent Gaussian Z-IC with state noncausally known at the transmitters, let $R_2^* = \frac{1}{2}\log(\frac{a^2P_2(P_2+b^2Q+1)}{P_2Q(ab-\beta)^2+a^2P_2+\beta^2Q})$, where $\beta = \frac{a^2P_2}{a^2P_2+P_1+1}$. If $R_2^* > \frac{1}{2}\log(1+\frac{a^2P_2}{1+P_1})$, then the line $B-B'$ are on the capacity region boundary with the rate coordinates of the points B and B' given by*

$$\begin{aligned} &\text{Point } B: \left(\frac{1}{2}\log(1+P_1), \frac{1}{2}\log(1+\frac{a^2P_2}{1+P_1})\right) \\ &\text{Point } B': \left(\frac{1}{2}\log(1+a^2P_2+P_1)-R_2^*, R_2^*\right). \end{aligned} \tag{26}$$

Proof. See Appendix F. □

4.3 Comparison of State-Dependent Regular IC and Z-IC

In this subsection, we compare the result in Theorem 3 for the state-dependent regular IC and the result in Theorem 4 for the state-dependent Z-IC in the strong interference regime.

In Fig. 5, we plot the parameter ranges characterized in Theorem 3 and in Theorem 4. For both the regular IC and the Z-IC, we set $P_1 = 1$, $P_2 = 1$, $Q = 2$ and $a = 1.2$. Moreover, for the regular IC, we set $b = 4$, which implies that the interference is strong enough such that its corresponding channel without state has the same capacity region as that of the Z-IC. Thus, the only flexible parameter left for both the regular IC and the Z-IC is the scaling coefficient c for the state. We study the range of c that guarantees the points on the line $B-E$ to be on the capacity region boundary of the state-dependent regular IC and Z-IC. We note that each point on the line $B-E$ can be parameterized as the rate pair $(R_1, R_2) = (R_1, \frac{1}{2}\log(1+P_1+a^2P_2)-R_1)$, where R_1 changes from $R_1 = 0.5$ (corresponding to point B) to $R_1 = \frac{1}{2}\log 1.72$ (corresponding to point E). In Fig. 5, for each R_1 (and hence for each corresponding point on the $B-E$ line), we plot the range of c that guarantees

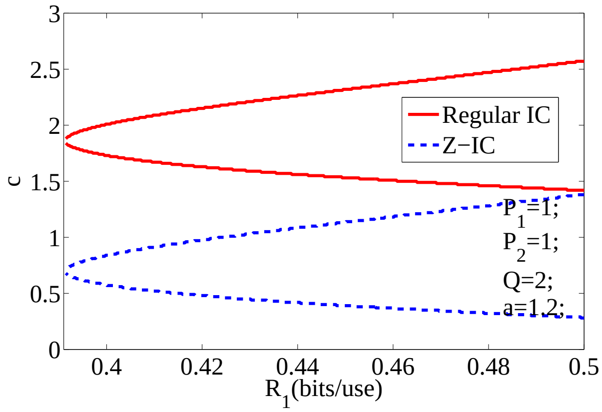

Figure 5: Ranges of c under which points on the sum capacity boundary of the strong regular/Z-IC without state can be achieved by the state-dependent channel

the point (R_1, R_2) to be on the capacity region boundary of the state-dependent regular IC to be between the two solid lines, and plot the range of c that guarantees the point (R_1, R_2) to be on the capacity region boundary of the state-dependent Z-IC between the two dashed lines. It can be seen that although the two ranges do not overlap, their structures are similar and the sizes of the ranges are comparable. This implies that both channels have the same flexibility to achieve the capacity region boundary point of the corresponding channel without state, and hence suggests that neither channel cancels the state more easily than the other. This is because for both the regular IC and the Z-IC, the layered dirty paper coding is designed in the same way to successively cancel the state for receiver 1. Hence, the advantage of the Z-IC at the other receiver is not significant due to the state interference that is not fully canceled. We further note that Fig. 5 also suggests that it is easier to achieve a point on the $B-E$ line when the point is closer to the point B for both channels.

5 Weak Interference Regime

In this section, we study both the state-dependent regular IC and Z-IC in the weak interference regime, in which the channel parameters satisfy $|a(1+b^2P_1)|+|b(1+a^2P_2)| \leqslant 1$ for the regular IC and satisfy $a^2 \leqslant 1$ for the Z-IC. Under such conditions, the sum capacity for the regular IC without state has been established in [15–17], and for the Z-IC without state has been established in [19]. In both cases, the sum capacity can be achieved by treating interference as noise at each receiver. Hence, for the corresponding state-dependent IC, independent dirty paper coding at two transmitters to cancel the state at their corresponding receivers (treating the interference as noise) can achieve the same sum capacity. Decoding at each receiver is not affected by how the interference signal is coded. Such an observation yields the following results.

Theorem 5. *For the state-dependent Gaussian IC with state noncausally known at both* *transmitters, if* $|a(1+b^2P_1)| + |b(1+a^2P_2)| \leqslant 1$*, then the sum capacity is given by*

$$C_{sum} = \frac{1}{2}\log\left(1+\frac{P_1}{a^2P_2+1}\right) + \frac{1}{2}\log\left(1+\frac{P_2}{b^2P_1+1}\right). \tag{27}$$

For the state-dependent Gaussian Z-IC with state noncausally known at both transmitters, if $a^2 \leqslant 1$*, then the sum capacity is given by*

$$C_{sum} = \frac{1}{2}\log\left(1+\frac{P_1}{a^2P_2+1}\right) + \frac{1}{2}\log\left(1+P_2\right). \tag{28}$$

Proof. See Appendix G. □

6 Conclusion

In this paper, we studied the two-user state-dependent Gaussian regular and Z-ICs with state noncausally known at both transmitters and unknown at either receiver. Our focus was on characterizing the conditions under which such state-dependent channel achieves the capacity of the corresponding channel without state. We showed that under certain conditions, dirty paper coding cooperatively designed between the two transmitters can achieve the capacity region in the very strong interference regime, and layered dirty paper coding and successive state cancelation can achieve points on the capacity region boundary in the strong regime. For the weak interference regime, cooperative dirty paper coding is not necessary to achieve the sum capacity due to the fact that treating interference as noise achieves the sum capacity for the channel without state. Our comparison between the state-dependent regular and Z-ICs suggests that, with the presence of state, interference can be advantageous in order for the two transmitters to cooperatively cancel the state at receivers. Future work will include generalization of the current study to multiuser channels, exploration of the scenarios when the state corruption at the two receivers are independent, and study of state-dependent multi-antenna ICs. We note that our model also captures scenarios in cognitive radio networks, where the state plays the role of the primary transmission, and is known by secondary users. It will thus be of interest to further incorporate the primary transmission into the system model.

Appendix

A Proof of Proposition 1

We use random codes and fix the following joint distribution:

$$P_{SUX_1VX_2Y_1Y_2} = P_S P_{U|S} P_{X_1|US} P_{V|S} P_{X_2|VS} P_{Y_1Y_2|X_1X_2S}.$$

Let $T_\epsilon^n(P_{SUX_1VX_2Y_1Y_2})$ denote the strongly joint ϵ-typical set based on the above distribution.

Code Construction:

1. Generate $2^{n(R_1+R_1')}$ codewords $U^n(w_1, l_1)$ with i.i.d. components based on P_U. Index these codewords by $w_1 = 1, \cdots, 2^{nR_1}$. $l_1 = 1, 2, \cdots, 2^{nR_1'}$.
2. Generate $2^{n(R_2+R_2')}$ codewords $V^n(w_2, l_2)$ with i.i.d. components based on P_V. Index these codewords by $w_2 = 1, \cdots, 2^{nR_2}$. $l_2 = 1, 2, \cdots, 2^{nR_2'}$.

Encoding:

1. Encoder 1: Given w_1, and s^n, select $u^n(w_1, \tilde{l}_1)$ such that
$$(u^n(w_1, \tilde{l}_1), s^n) \in T_\epsilon^n(P_{US}).$$
Otherwise, set $\tilde{l}_1 = 1$. It can be shown that for large n, such u^n exists with high probability if
$$R_1' > I(U; S). \tag{29}$$
Given selected $u^n(w_1, \tilde{l}_1)$ and s^n, generate x_1^n with i.i.d. components based on $P_{X_1|US}$ for transmission.
2. Encoder 2: Given w_2, and s^n, select $v^n(w_2, \tilde{l}_2)$ such that
$$(v^n(w_2, \tilde{l}_2), s^n) \in T_\epsilon^n(P_{VS}).$$
Otherwise, set $\tilde{l}_2 = 1$. It can be shown that for large n, such v^n exists with high probability if
$$R_2' > I(V; S). \tag{30}$$
Given selected $v^n(w_2, \tilde{l}_2)$ and s^n, generate x_2^n with i.i.d. components based on $P_{X_2|VS}$ for transmission.

Decoding:

1 Decoder 1: Given y_1^n, find the unique pair $(\hat{w}_2, \hat{l}_2)$ such that
$$(v^n(\hat{w}_2, \hat{l}_2), y_1^n) \in T_\epsilon^n(P_{VY_1}).$$
If no or more than one such pairs $(\hat{w}_2, \hat{l}_2)$ can be found, then declare error. One can show that for sufficiently large n, decoding is correct with high probability if
$$R_2 + R_2' \leqslant I(V; Y_1). \tag{31}$$
After successfully decoding v^n, find the unique pair $(\hat{w}_1, \hat{l}_1)$ such that
$$(v^n(\hat{w}_2, \hat{l}_2), u^n(\hat{w}_1, \hat{l}_1), y_1^n) \in T_\epsilon^n(P_{VUY_1}).$$
If no or more than one such pairs with different w_1 can be found, then declare error. One can show that for sufficiently large n, decoding is correct with high probability if
$$R_1 + R_1' \leqslant I(U; VY_1). \tag{32}$$

2. Decoder 2: Given y_2^n, find the unique pair $(\hat{w}_1, \hat{l}_1)$ such that

$$(u^n(\hat{w}_1, \hat{l}_1), y_2^n) \in T_\epsilon^n(P_{UY_2}).$$

If no or more than one such pairs $(\hat{w}_1, \hat{l}_1)$ can be found, then declare error. One can show that for sufficiently large n, decoding is correct with high probability if

$$R_1 + R_1' \leqslant I(U; Y_2). \tag{33}$$

After successfully decoding u^n, find the unique pair $(\hat{w}_2, \hat{l}_2)$ such that

$$(u^n(\hat{w}_1, \hat{l}_1), v^n(\hat{w}_2, \hat{l}_2), y_2^n) \in T_\epsilon^n(P_{UVY_2}).$$

If no or more than one such pairs with different w_2 can be found, then declare error. One can show that for sufficiently large n, decoding is correct with high probability if

$$R_2 + R_2' \leqslant I(V; UY_2). \tag{34}$$

Proposition 1 is thus proved by combining (29)-(34).

B Proof of Proposition 2

The coding scheme for the very strong Z-IC is similar to that for the regular IC. More specifically, codebook generation, encoding and decoding for decoder 1 are the same as those in Appendix A. We next describe decoding for decoder 2 as follows.

Decoding for decoder 2:

Given y_2^n, find the unique pair $(\hat{w}_2, \hat{l}_2)$ such that

$$(v^n(\hat{w}_2, \hat{l}_2), y_2^n) \in T_\epsilon^n(P_{VY_2}).$$

If no or more than one such pairs with different w_2 can be found, then declare error. One can show that for sufficiently large n, decoding is correct with high probability if

$$R_2 + R_2' \leqslant I(V; Y_2).$$

If $I(V; Y_2) \leqslant I(V; Y_1)$, then the bound $R_2 + R_2' \leqslant I(V; Y_1)$ obtained in decoding for decoder 1 (see (31)) is redundant. Hence, the corresponding achievable region is as given in Proposition 2.

C Proof of Proposition 3

The achievable scheme applies rate splitting, superposition and Gel'fand-Pinsker binning. In particular, we split the message W_1 into two components W_{11} and W_{12}, and split W_2 into two components W_{21} and W_{22}. We use random codes and fix the following joint distribution:

$$P_{SU_1U_2X_1V_1V_2X_2Y_1Y_2} = P_S P_{U_1U_2|S} P_{X_1|U_1U_2S} P_{V_1V_2|S} P_{X_2|V_1V_2S} P_{Y_1Y_2|X_1X_2S}.$$

Code Construction:

1. Generate $2^{n(R_{11}+R'_{11})}$ codewords $U_1^n(w_{11}, l_{11})$ with i.i.d. components based on P_{U_1}. Index these codewords by $w_{11} = 1,\cdots, 2^{nR_{11}}$, $l_{11} = 1, 2,\cdots, 2^{nR'_{11}}$.
2. For each $u_1^n(w_{11}, l_{11})$, generate $2^{n(R_{12}+R'_{12})}$ codewords $U_2^n(w_{11}, l_{11}, w_{12}, l_{12})$ with i.i.d. components based on $P_{U_2|U_1}$. Index these codewords by $w_{12} = 1,\cdots, 2^{nR_{12}}$, $l_{12} = 1, 2,\cdots, 2^{nR'_{12}}$.
3. Generate $2^{n(R_{21}+R'_{21})}$ codewords $V_1^n(w_{21}, l_{21})$ with i.i.d. components based on P_{V_1}. Index these codewords by $w_{21} = 1,\cdots, 2^{nR_{21}}$, $l_{21} = 1, 2,\cdots, 2^{nR'_{21}}$.
4. For each $v_1^n(w_{21}, l_{21})$, generate $2^{n(R_{22}+R'_{22})}$ codewords $V_2^n(w_{21}, l_{21}, w_{22}, l_{22})$ with i.i.d. components based on $P_{V_2|V_1}$, Index these codewords by $w_{22} = 1,\cdots, 2^{nR_{22}}$, $l_{22} = 1, 2,\cdots, 2^{nR'_{22}}$.

Encoding:

1. Encoder 1: Given w_{11}, and s^n, select $u_1^n(w_{11}, \tilde{l}_{11})$ such that

   $$(u_1^n(w_{11}, \tilde{l}_{11}), s^n) \in T_\epsilon^n(P_{U_1S}).$$

   Otherwise, set $\tilde{l}_{11} = 1$. It can be shown that for large n, such u_1^n exists with high probability if
   $$R'_{11} > I(U_1; S). \tag{35}$$

   Given w_{12}, $w_{11}, \tilde{l}_{11}$, and s^n, select $u_2^n(w_{11}, \tilde{l}_{11}, w_{12}, \tilde{l}_{12})$ such that

   $$(u_2^n(w_{11}, \tilde{l}_{11}, w_{12}, \tilde{l}_{12}), u_1^n(w_{11}, \tilde{l}_{11}), s^n) \in T_\epsilon^n(P_{U_2SU_1}).$$

   Otherwise, set $\tilde{l}_{12} = 1$. It can be shown that for large n, such u_2^n exists with high probability if
   $$R'_{12} > I(U_2; S|U_1). \tag{36}$$

   Given $u_1^n(w_{11}, \tilde{l}_{11})$, $u_2^n(w_{11}, \tilde{l}_{11}, w_{12}, \tilde{l}_{12})$, and s^n, generate x_1^n with i.i.d. components based on $P_{X_1|U_1U_2S}$ for transmission.
2. Encoder 2: Given w_{21}, and s^n, select $v_1^n(w_{21}, \tilde{l}_{21})$ such that

   $$(v_1^n(w_{21}, \tilde{l}_{21}), s^n) \in T_\epsilon^n(P_{V_1S}).$$

Otherwise, set $\tilde{l}_{21} = 1$. It can be shown that for large n, such v_1^n exists with high probability if

$$R'_{21} > I(V_1; S). \tag{37}$$

Given w_{22}, $w_{21}, \tilde{l}_{21}$ and s^n, select $v_2^n(w_{21}, \tilde{l}_{21}, w_{22}, \tilde{l}_{22})$ such that

$$(v_2^n(w_{21}, \tilde{l}_{21}, w_{22}, \tilde{l}_{22}), v_1^n(w_{21}, \tilde{l}_{21}), s^n) \in T_\epsilon^n(P_{V_2 S V_1}).$$

Otherwise, set $\tilde{l}_{22} = 1$. It can be shown that for large n, such v_2^n exists with high probability if

$$R'_{22} > I(V_2; S|V_1). \tag{38}$$

Given $v_1^n(w_{21}, \tilde{l}_{21})$, $v_2^n(w_{21}, \tilde{l}_{21}, w_{22}, \tilde{l}_{22})$ and s^n, generate x_2^n with i.i.d. components based on $P_{X_2|V_1 V_2 S}$ for transmission.

Decoding:

1. Decoder 1: Given y_1^n, find the unique pair $(\hat{w}_{11}, \hat{l}_{11})$ such that

$$(u_1^n(\hat{w}_{11}, \hat{l}_{11}), y_1^n) \in T_\epsilon^n(P_{U_1 Y_1}).$$

If no or more than one such pairs with different w_{11} can be found, then declare error. One can show that for sufficiently large n, decoding is correct with high probability if

$$R_{11} + R'_{11} \leqslant I(U_1; Y_1).$$

After successfully decoding u_1^n, find the unique tuple $(\hat{w}_{21}, \hat{l}_{21}, \hat{w}_{22}, \hat{l}_{22})$ such that

$$(v_1^n(\hat{w}_{21}, \hat{l}_{21}), v_2^n(\hat{w}_{21}, \hat{l}_{21}, \hat{w}_{22}, \hat{l}_{22}), u_1^n(\hat{w}_{11}, \hat{l}_{11}), y_1^n) \in T_\epsilon^n(P_{V_1 V_2 U_1 Y_1}).$$

If no or more than one such tuples with different rate pairs (w_{21}, w_{22}) can be found, then declare error. One can show that for sufficiently large n, decoding is correct with high probability if

$$\begin{aligned} R_{21} + R'_{21} &\leqslant I(V_1; U_1 Y_1) \\ R_{22} + R'_{22} &\leqslant I(V_2; U_1 Y_1 | V_1) \end{aligned}$$

After successfully decoding u_1^n, v_1^n and v_2^n, we find the unique pair $(\hat{w}_{12}, \hat{l}_{12})$ such that

$$(u_2^n(\hat{w}_{11}, \hat{l}_{11}, \hat{w}_{12}, \hat{l}_{12}), v_1^n(\hat{w}_{21}, \hat{l}_{21}), v_2^n(\hat{w}_{21}, \hat{l}_{21}, \hat{w}_{22}, \hat{l}_{22}), u_1^n(\hat{w}_{11}, \hat{l}_{11}), y_1^n) \in T_\epsilon^n(P_{U_2 U_1 V_1 V_2 Y_1}).$$

If no or more than one such pair with different w_{12} can be found, then declare error. One can show that for sufficiently large n, decoding is correct with high probability if

$$R_{12} + R'_{12} \leqslant I(U_2; V_1 V_2 Y_1 | U_1) \tag{39}$$

2. Decoder 2: Given y_2^n, find the unique pair $(\hat{w}_{21}, \hat{l}_{21})$ such that

$$(v_1^n(\hat{w}_{21}, \hat{l}_{21}), y_2^n) \in T_\epsilon^n(P_{V_1Y_2}).$$

If no or more than one such pairs with different w_{21} can be found, then declare error. One can show that for sufficiently large n, decoding is correct with high probability if

$$R_{21} + R'_{21} \leqslant I(V_1; Y_2).$$

After successfully decoding v_1^n, find the unique tuple $(\hat{w}_{11}, \hat{l}_{11}, \hat{w}_{12}, \hat{l}_{12})$ such that

$$(u_1^n(\hat{w}_{11}, \hat{l}_{11}), u_2^n(\hat{w}_{11}, \hat{l}_{11}, \hat{w}_{12}, \hat{l}_{12}), v_1^n(\hat{w}_{21}, \hat{l}_{21}), y_2^n) \in T_\epsilon^n(P_{U_1U_2V_1Y_2}).$$

If no or more than one such tuples with different rate pair (w_{11}, w_{12}) can be found, then declare error. One can show that for sufficiently large n, decoding is correct with high probability if

$$\begin{aligned} R_{11} + R'_{11} &\leqslant I(U_1; V_1Y_2) \\ R_{12} + R'_{12} &\leqslant I(U_2; V_1Y_2|U_1) \end{aligned}$$

After successfully decoding v_1^n, u_1^n and u_2^n, we find the unique pair $(\hat{w}_{22}, \hat{l}_{22})$ such that

$$(v_2^n(\hat{w}_{21}, \hat{l}_{21}, \hat{w}_{22}, \hat{l}_{22}), u_1^n(\hat{w}_{11}, \hat{l}_{11}), u_2^n(\hat{w}_{11}, \hat{l}_{11}, \hat{w}_{12}, \hat{l}_{12}), v_1^n(\hat{w}_{21}, \hat{l}_{21}), y_2^n) \in T_\epsilon^n(P_{V_2V_1U_1U_2Y_2}).$$

If no or more than one such pair with different w_{22} can be found, then declare error. One can show that for sufficiently large n, decoding is correct with high probability if

$$R_{22} + R'_{22} \leqslant I(V_2; U_1U_2Y_2|V_1) \tag{40}$$

The corresponding achievable region is thus characterized by

$$\begin{aligned} R_{11} &\leqslant \min\{I(U_1; Y_1), I(U_1; V_1Y_2)\} - I(U_1; S) \\ R_{12} &\leqslant \min\{I(U_2; V_1V_2Y_1|U_1), I(U_2; V_1Y_2|U_1)\} - I(U_2; S|U_1) \\ R_{21} &\leqslant \min\{I(V_1; Y_2), I(V_1; U_1Y_1)\} - I(V_1; S) \\ R_{22} &\leqslant \min\{I(V_2; U_1U_2Y_2|V_1), I(V_2; U_1Y_1|V_1)\} - I(V_2; S|V_1) \end{aligned}$$

Proposition 3 follows by setting $R_1 = R_{11} + R_{12}$ and $R_2 = R_{21} + R_{22}$, and applying Fourier-Motzkin elimination to the above region.

D Proof of Proposition 4

The coding scheme for the strong Z-IC is similar to that for the regular IC. More specifically, codebook generation and encoding for the strong Z-IC are the same as those for the regular IC provided in Appendix C. We next describe decoding as follows.

Decoding:

1. Decoder 1: Given y_1^n, find the unique pair $(\hat{w}_{21}, \hat{l}_{21})$ such that

$$(v_1^n(\hat{w}_{21}, \hat{l}_{21}), y_1^n) \in T_\epsilon^n(P_{V_1 Y_1}).$$

If no or more than one such pairs with different w_{21} can be found, then declare error. One can show that for sufficiently large n, decoding is correct with high probability if

$$R_{21} + R'_{21} \leqslant I(V_1; Y_1).$$

After successfully decoding v_1^n, find the unique pair $(\hat{w}_{11}, \hat{l}_{11})$ such that

$$(v_1^n(\hat{w}_{21}, \hat{l}_{21}), u_1^n(\hat{w}_{11}, \hat{l}_{11}), y_1^n) \in T_\epsilon^n(P_{V_1 U_1 Y_1}).$$

If no or more than one such rate pairs with different w_{11} can be found, then declare error. One can show that for sufficiently large n, decoding is correct with high probability if

$$R_{11} + R'_{11} \leqslant I(U_1; V_1 Y_1).$$

After successfully decoding u_1^n and v_1^n we find the unique pair $(\hat{w}_{22}, \hat{l}_{22})$ such that

$$(v_2^n(\hat{w}_{21}, \hat{l}_{21}, \hat{w}_{22}, \hat{l}_{22}), v_1^n(\hat{w}_{21}, \hat{l}_{21}), u_1^n(\hat{w}_{11}, \hat{l}_{11}), y_1^n) \in T_\epsilon^n(P_{U_1 V_1 V_2 Y_1}).$$

If no or more than one such pairs with different w_{22} can be found, then declare error. One can show that for sufficiently large n, decoding is correct with high probability if

$$R_{22} + R'_{22} \leqslant I(V_2; U_1 Y_1 | V_1) \tag{41}$$

After successfully decoding u_1^n, v_1^n and v_2^n, we find the unique pair $(\hat{w}_{12}, \hat{l}_{12})$ such that

$$(u_2^n(\hat{w}_{11}, \hat{l}_{11}, \hat{w}_{12}, \hat{l}_{12}), v_1^n(\hat{w}_{21}, \hat{l}_{21}), v_2^n(\hat{w}_{21}, \hat{l}_{21}, \hat{w}_{22}, \hat{l}_{22}), u_1^n(\hat{w}_{11}, \hat{l}_{11}), y_1^n) \in T_\epsilon^n(P_{U_2 U_1 V_1 V_2 Y_1}).$$

If no or more than one such pairs with different w_{12} can be found, then declare error. One can show that for sufficiently large n, decoding is correct with high probability if

$$R_{12} + R'_{12} \leqslant I(U_2; V_1 V_2 Y_1 | U_1). \tag{42}$$

2. Decoder 2: Given y_2^n, find the unique pair $(\hat{w}_{21}, \hat{l}_{21})$ such that

$$(v_1^n(\hat{w}_{21}, \hat{l}_{21}), y_2^n) \in T_\epsilon^n(P_{V_1Y_2}).$$

If no or more than one such pairs with different w_{21} can be found, then declare error. One can show that for sufficiently large n, decoding is correct with high probability if

$$R_{21} + R'_{21} \leqslant I(V_1; Y_2).$$

After successfully decoding v_1^n, find the unique pair $(\hat{w}_{22}, \hat{l}_{22})$ such that

$$(v_2^n(\hat{w}_{21}, \hat{l}_{21}, \hat{w}_{22}, \hat{l}_{22}), v_1^n(\hat{w}_{21}, \hat{l}_{21}), y_2^n) \in T_\epsilon^n(P_{V_2V_1Y_2}).$$

If no or more than one such pairs with different w_{22} can be found, then declare error. One can show that for sufficiently large n, decoding is correct with high probability if

$$R_{22} + R'_{22} \leqslant I(V_2; Y_2|V_1). \tag{43}$$

The corresponding achievable region is thus characterized by

$$\begin{aligned}
&R_{11} \leqslant I(U_1; Y_1V_1) - I(U_1; S)\\
&R_{12} \leqslant I(U_2; V_1V_2Y_1|U_1) - I(U_2; S|U_1)\\
&R_{21} \leqslant \min\{I(V_1; Y_2), I(V_1; Y_1)\} - I(V_1; S)\\
&R_{22} \leqslant \min\{I(V_2; Y_2|V_1), I(V_2; U_1Y_1|V_1)\} - I(V_2; S|V_1).
\end{aligned}$$

Proposition 4 then follows by setting $R_1 = R_{11} + R_{12}$ and $R_2 = R_{21} + R_{22}$, and applying Fourier-Motzkin elimination to the above region.

E Proof of Proposition 5

Assume $P'_{1B'}$, $P''_{1B'}$, $P'_{2B'}$ and $P''_{2B'}$ are power allocation parameters corresponding to the given point B' under which the conditions in (24a) and (24b) are satisfied. In order to prove that the point B is also achievable, we design the following coding scheme. We split W_1 into W_{11} and W_{12}, and split W_2 into W_{21} and W_{22}. We then encode the messages W_{11}, W_{12}, W_{21} and W_{22} into auxiliary random variables U_1, U_2, V_1, and V_2, respectively. Then receiver 1 decodes in the order of V_1, V_2, U_1 and U_2, and receiver 2 decodes in the order of V_1 and V_2. It can be shown that (R_1, R_2) is achievable if it satisfies

$$\begin{aligned}
R_1 &\leqslant I(U_1; Y_1V_1V_2) + I(U_2; V_1V_2Y_1|U_1) - I(U_1, U_2; S)\\
R_2 &\leqslant \min\{I(V_1; Y_2), I(V_1; Y_1)\}\\
&\quad + \min\{I(V_2; Y_2|V_1), I(V_2; Y_1|V_1)\} - I(V_1V_2; S)
\end{aligned} \tag{44}$$

for some distribution $P_{SU_1U_2V_1V_2X_2X_1Y_2Y_1} = P_S P_{U_1U_2|S} P_{V_1V_2|S} P_{X_1|U_1U_2S} P_{X_2|V_1V_2S} P_{Y_1|SX_1X_2} P_{Y_2|SX_2}$. We now compute (44) by setting the auxiliary random variables as in (19), with the power allocations $P'_{1B'}$, $P''_{1B'}$, $P'_{2B'}$ and $P''_{2B'}$ for X'_1, X''_1, X'_2 and X''_2 in U_1, U_2, V_1 and V_2, respectively. It can be verified that due to (24a) and (24b) that the power allocation parameters satisfy, the two mutual information terms $I(V_1;Y_2)$ and $I(V_2;Y_2|V_1)$ in R_2 become redundant. It can then be verified that the rate pair corresponding to the point B satisfies the resulting (44), and is hence achievable. Thus, the line $B-B'$ is achievable by time sharing.

F Proof of Corollary 2

It is sufficient to show that the point B' satisfies Theorem 4, i.e., it is on the capacity region boundary. Then following Proposition 5, the line $B-B'$ is on the capacity region boundary. It can be verified that the point B' is characterized by (17) by setting $P'_2=0$, $P''_2=P_2$, P''_1 to satisfy

$$1+\frac{a^2P_2}{P''_1+1}\leqslant\frac{a^2P_2(P_2+b^2Q+1)}{P_2Q(ab-\beta)^2+a^2P_2+\beta^2Q}, \tag{45}$$

and $P'_1=P_1-P''_1$. Then it can be verified that the condition (24a) and (24b) in Theorem 4 are satisfied by the point B'.

G Proof of Theorem 5

Similarly to [15], [16] and [17], to achieve the sum capacity for the state-dependent Gaussian IC, we apply dirty paper coding for X_1 treating aX_2+N_1 as noise and apply dirty paper coding for X_2 treating bX_1+N_1 as noise. Thus, the point $(R_1,R_2)=(\frac{1}{2}\log(1+\frac{P_1}{a^2P_2+1}),\frac{1}{2}\log(1+\frac{P_2}{b^2P_1+1}))$ can be achieved.

For the outer bound, applying Fano's inequality, we have

$$\begin{aligned}
nR_1 &\leqslant I(W_1;Y_1^n)+n\epsilon_n\\
&\leqslant I(W_1;Y_1^nS^n)+n\epsilon_n\\
&= I(W_1;Y_1^n|S^n)+n\epsilon_n\\
&\leqslant I(W_1X_1^n;Y_1^n|S^n)+n\epsilon_n\\
&= I(X_1^n;Y_1^n|S^n)+I(W_1;Y_1^n|S^nX_1^n)+n\epsilon_n\\
&= I(X_1^n;Y_1^n|S^n)+n\epsilon_n\\
&= I(X_1^n;X_1^n+aX_2^n+S^n+N_1^n|S^n)+n\epsilon_n\\
&= I(X_1^n;X_1^n+aX_2^n+N_1^n|S^n)+n\epsilon_n\\
&= \sum_{s^n}p(S^n=s^n)I(X_1^n;X_1^n+aX_2^n+N_1^n|S^n=s^n)+n\epsilon_n.
\end{aligned} \tag{46}$$

Similarly, we have

$$nR_2\leqslant\sum_{s^n}p(S^n=s^n)I(X_2^n;bX_1^n+X_2^n+N_2^n|S^n=s^n)+n\epsilon_n. \tag{47}$$

Combining (46) and (47), we obtain

$$\begin{aligned} n(R_1+R_2) &\leqslant \sum_{s^n} p(S^n=s^n) \max_{P_{X_1^n|S^n}P_{X_2^n|S^n}} [I(X_1^n; X_1^n + aX_2^n + N_1^n | S^n = s^n) \\ &\qquad\qquad + I(X_2^n; bX_1^n + X_2^n + N_2^n | S^n = s^n)] + 2n\epsilon_n \\ &= \sum_{s^n} p(S^n=s^n) \max_{P_{X_1^n}P_{X_2^n}} [I(X_1^n; X_1^n + aX_2^n + N_1^n) + I(X_2^n; bX_1^n + X_2^n + N_2^n)] + 2n\epsilon_n \\ &= \max_{P_{X_1^n}P_{X_2^n}} [I(X_1^n; X_1^n + aX_2^n + N_1^n) + I(X_2^n; bX_1^n + X_2^n + N_2^n)] + 2n\epsilon_n. \end{aligned}$$

If $|a(1+b^2P_1)| + |b(1+a^2P_2)| \leqslant 1$, following the results in [16, Section IV.C], we further obtain

$$R_1 + R_2 \leqslant \frac{1}{2}\log\left(1+\frac{P_1}{a^2P_2+1}\right) + \frac{1}{2}\log\left(1+\frac{P_2}{b^2P_1+1}\right) + 2\epsilon_n.$$

Hence, the rate point $(R_1, R_2) = (\frac{1}{2}\log(1+\frac{P_1}{a^2P_2+1}), \frac{1}{2}\log(1+\frac{P_2}{b^2P_1+1}))$ is sum-rate optimal. Thus, the sum capacity is obtained.

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