

On the Capacity Bounds for Poisson Interference Channels

Lifeng Lai, Yingbin Liang and Shlomo Shamai (Shitz)

Abstract

The Poisson interference channel, which models optical communication systems with multiple transceivers, is investigated. Conditions for the strong interference regime are characterized and the corresponding capacity region is derived, which is the same as that of the compound Poisson multiple access channel with each receiver decoding both messages. For the cases when the strong interference conditions are not satisfied, inner and outer bounds on the capacity region are derived. The inner bound is derived via approximating the Poisson interference channel by a binary interference channel and then evaluating the corresponding Han-Kobayashi region. The outer bounds are obtained via various techniques including noise reduction, genie-aided scheme and degraded broadcast channel conversion. The Poisson Z-interference channel is then studied. The sum rate capacity is obtained when the cross link coefficient is either sufficiently small or sufficiently large.

Index Terms: Capacity region, Interference channel, Poisson channel, Sum rate capacity, Z-interference channel

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I. INTRODUCTION

The Poisson channel models optical communications when the receivers employ photon-sensitive devices to record the arrival of photons. The point-to-point Poisson channel has been intensively studied [2]–[5]. However, the study of multiuser Poisson channels is limited, with few exceptions on the multiple-access channel (MAC) [6] and on the broadcast channel [7]. The optical interference channel was discussed in [8], in which several open questions were posted. In this paper, we study the Poisson interference channel that models simultaneous transmission of multiple optical communication links. We note that another popular model for optical communications assumes that the background noise is Gaussian distributed [9], which is not treated in the current paper.

The Poisson interference channel differs from the standard Gaussian interference channel well-studied in, e.g., [10]–[13], in the following aspects. Unlike the Gaussian channel, the Poisson channel is not scale-invariant. Hence, one cannot arbitrarily multiply the input signal with a constant and later on divide the output with the same constant while preserving the mutual information between the input and the output. Furthermore, in the Gaussian interference channel, once the receiver decodes a signal, it can completely cancel the effects of this signal. This is not the case for the Poisson channel anymore. As a result, certain techniques used to derive various capacity bounds for the Gaussian interference channel are not applicable for the Poisson interference channel. On the other hand, the Poisson channel still has the properties of infinite divisibility and independent increments that the Gaussian channel has [14]. This similarity allows us to apply certain ideas designed for the Gaussian interference channel to the Poisson interference channel with appropriate modifications.

In this paper, we first identify the conditions for the strong interference regime and characterize the corresponding capacity region, which is the same as that of the compound Poisson MAC with each receiver decoding both messages. The basic idea involves carefully constructing a new Poisson process based on the decoded signal to enhance the receivers and then performing a proper thinning operation, in which we randomly and independently erase points from the Poisson process. On the other hand, we show that there is no corresponding very strong interference regime as that of the Gaussian interference channel, mainly due to the inability to completely cancel the effects of a known signal at the output of the Poisson channel.

We then study the case in which the strong interference conditions are not satisfied. We derive an inner bound on the capacity region by first approximating the Poisson interference channel with a binary interference channel via the approach introduced by Wyner [2], and then evaluating the Han-Kobayashi [15] region for the binary interference channel under certain input distributions. Furthermore, we derive several outer bounds on the capacity region. These bounds are derived based on reducing noise level at the receiver, providing various side-information to receivers, and connecting the channel to a degraded broadcast channel.

We further study the Poisson Z-interference channel, in which one of the cross link coefficients is zero. For this channel, we determine the sum rate capacity when the remaining cross link coefficient is either sufficiently small or sufficient large. We show that when the remaining cross link coefficient is sufficiently small, the sum capacity is achieved by treating the interference (if any) as noise. In this case, to maximize the sum rate capacity, the transmitter whose corresponding receiver does not experience the interference does not necessarily transmit at the maximum possible rate. This is in contrast to the corresponding case of the Gaussian Z-interference channel, in which the user not experiencing the interference always transmits at the full rate in order to achieve the sum rate capacity. When the cross link coefficient is sufficiently large, the sum capacity is achieved by letting the receiver experiencing interference decode both messages.

The remainder of the paper is organized as follows. In Section II, we introduce the Poisson interference channel model. In Section III, we derive conditions for the strong interference channel. We derive an inner bound for the capacity region for the general Poisson interference channel in Section IV. Several outer bounds are derived in Section V. In Section VI, we study the sum capacity for the Poisson Z-interference channel. Finally, we conclude the paper with a few remarks in Section VII.

II. SYSTEM MODEL

A. Poisson Interference Channel

We study the two-user Poisson interference channel, in which inputs $X_1(t)$ and $X_2(t)$ from the transmitters satisfy the following conditions:

$$0 \leq X_i(t) \leq A_i, \text{ for all } t \text{ and for } i = 1, 2, \quad (1)$$

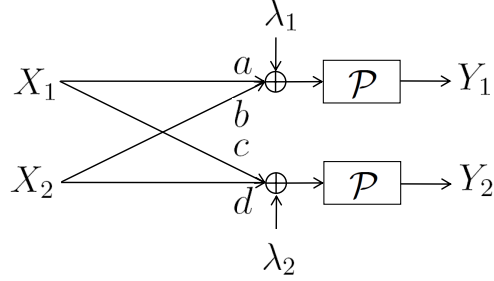


Fig. 1: The Poisson interference channel

which means that the input of the Poisson channel is nonnegative and less than a threshold due to safety considerations¹. The output $Y_1(t)$ for $0 \leq t \leq T$ at receiver 1 is a doubly stochastic Poisson process with the instantaneous rate $aX_1(t) + bX_2(t) + \lambda_1$, and the output $Y_2(t)$ for $0 \leq t \leq T$ at receiver 2 is another doubly stochastic Poisson process with the instantaneous rate $cX_1(t) + dX_2(t) + \lambda_2$. More precisely, for all $0 \leq t < s \leq T$,

$$\Pr \{Y_1(s) - Y_1(t) = k | X_1(0, T), X_2(0, T)\} = \frac{1}{k!} \Lambda_1^k e^{-\Lambda_1}, k = 0, 1, \dots,$$

$$\Pr \{Y_2(s) - Y_2(t) = k | X_1(0, T), X_2(0, T)\} = \frac{1}{k!} \Lambda_2^k e^{-\Lambda_2}, k = 0, 1, \dots,$$

in which

$$\Lambda_1 = \int_t^s (aX_1(\tau) + bX_2(\tau) + \lambda_1) d\tau,$$

$$\Lambda_2 = \int_t^s (cX_1(\tau) + dX_2(\tau) + \lambda_2) d\tau, \quad (2)$$

and $X_i(0, T)$ denotes $\{X_i(t) : 0 \leq t \leq T\}$. We use $Y_1 = \mathcal{P}(aX_1(0, T) + bX_2(0, T) + \lambda_1)$ and $Y_2 = \mathcal{P}(cX_1(0, T) + dX_2(0, T) + \lambda_2)$ to denote the output processes at receivers 1 and 2, respectively. Figure 1 illustrates the Poisson interference channel under consideration. It is easy to see that the received signals at receivers are non-decreasing step functions.

We use $M_i \in \mathcal{M}_i$ to denote the message of transmitter i . A code $(T, \mathcal{M}_1, \mathcal{M}_2, \mu_1, \mu_2)$ is defined to include

- 1) Two message sets $\mathcal{M}_i = \{1, \dots, |\mathcal{M}_i|\}$ for $i = 1, 2$;

¹In this paper, only peak power constraint is considered. Certain results can be extended to the case with additional average power constraints.

- 2) Two sets of waveforms $X_{i,m_i}(\cdot)$, for $m_i \in \{1, \dots, |\mathcal{M}_i|\}$ and $i = 1, 2$, each of which satisfies the power constraint (1);
- 3) Two encoders, f_i at encoder i for $i = 1, 2$, each of which maps each message m_i to a waveform $x_{i,m_i}(\cdot)$;
- 4) Two decoders, D_i at receiver i for $i = 1, 2$: $Y_i(0, T) \rightarrow \hat{M}_i \in \{1, \dots, |\mathcal{M}_i|\}$, each of which maps a received signal $Y_i(0, T)$ to a message \hat{m}_i ;
- 5) Two average error probabilities: $\mu_i = \Pr\{\hat{M}_i \neq M_i\}$ for $i = 1, 2$.

A rate pair (R_1, R_2) is said to be achievable if for any $\epsilon > 0$, there exists a sequence of codes $(T, \mathcal{M}_1, \mathcal{M}_2, \mu_1, \mu_2)$ with $|\mathcal{M}_i| \geq 2^{TR_i}$ and $\mu_i \leq \epsilon$ for $i = 1, 2$. The capacity region \mathcal{C} is the closure of the set of all achievable rate pairs. We are also interested in the sum rate capacity defined as

$$C_{sum} = \max_{(R_1, R_2) \in \mathcal{C}} \{R_1 + R_2\}. \quad (3)$$

In order to simplify the notation, we define $\bar{p} = 1 - p$ for $0 \leq p \leq 1$ and define the following functions that are used throughout the paper:

$$\psi(\alpha, \beta; \gamma, \lambda) := (\alpha + \beta + \lambda) \ln(\alpha + \beta + \lambda) - (\gamma + \beta + \lambda) \ln(\gamma + \beta + \lambda), \quad (4)$$

$$\phi(p, A, \lambda) := \bar{p}\lambda \ln \lambda + p(A + \lambda) \ln(A + \lambda) - (Ap + \lambda) \ln(Ap + \lambda). \quad (5)$$

In addition, we use

$$\tilde{Y} = Y \downarrow p \quad (6)$$

to denote the thinning operation with the parameter p , in which we obtain a process \tilde{Y} by randomly and independently erasing points from a Poisson process Y with the probability \bar{p} (thus keeping each point of Y with the probability p).

B. Poisson Z-interference Channel

In this paper, we will also consider the Poisson Z-interference channel shown in Figure 2. In the Poisson Z-interference channel, one of the receivers (say receiver 1) does not experience any interference. One can get the Poisson Z-interference channel model by setting $b = 0$ in the Poisson interference channel discussed in Section II-A. For the Poisson Z-interference channel, we will focus on the sum rate capacity.

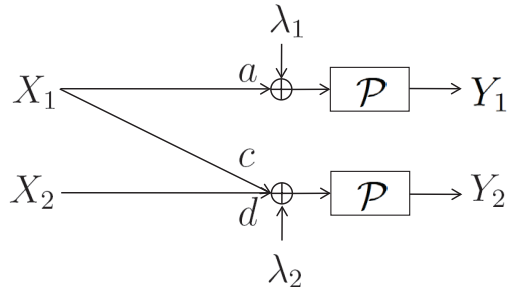


Fig. 2: The Poisson Z-interference Channel

C. Differences Between Poisson and Gaussian Interference Channels

In the following, we summarize the differences between the Poisson and Gaussian interference channels, which helps to explain why the analysis of the Poisson interference channel requires development of new techniques.

There are two main differences between the Poisson interference channel and the Gaussian interference channel. Firstly, the Poisson channel is not scale-invariant. More specifically, for a constant $a \neq 1$, $a\mathcal{P}(X_1(0, T) + \lambda_1/a)$ is different from $\mathcal{P}(aX_1(0, T) + \lambda_1)$, and as a result, $I(X_1(0, T); a\mathcal{P}(X_1(0, T) + \lambda_1/a))$ is different from $I(X_1(0, T); \mathcal{P}(aX_1(0, T) + \lambda_1))$. Secondly, in the Poisson interference channel, even if $X_1(0, T)$ is known, it is not possible to construct $\mathcal{P}(bX_2(0, T) + \lambda_1)$ from $\mathcal{P}(aX_1(0, T) + bX_2(0, T) + \lambda_1)$, i.e., it is not possible to completely cancel the effects of $X_1(0, T)$ from the output even if its value is known. Hence, complete interference cancellation is not possible in the Poisson interference channel. On the other hand, the Poisson channel also has the infinite divisibility and independent increments properties that the Gaussian channel has. This similarity allows us to apply certain ideas from the Gaussian interference channels to the Poisson interference channel.

III. CAPACITY REGION FOR THE STRONG INTERFERENCE CHANNEL

In this section, we characterize the capacity region of the Poisson interference channel in the so-called strong interference regime. We obtain the capacity region result by showing that if the channel parameters satisfy certain conditions, the capacity region of the Poisson interference channel is the same as that of a compound Poisson MAC. The central argument of our proof is to show that, if certain conditions are satisfied, any rate pair (R_1, R_2) achievable for the Poisson

interference channel is also achievable for the Poisson compound MAC. Since it is clear that rate pairs achievable for the compound MAC is achievable for the interference channel, the equivalence of the two capacity regions is established. The argument for the equivalence of these two channels is slightly different from that of the Gaussian interference channel [11], due to the differences between the Gaussian channel and the Poisson channel discussed in Section II. As detailed in the proof, it involves enhancement and thinning arguments.

Theorem 3.1: If

$$\frac{b}{d} \geq \frac{\lambda_1}{\lambda_2} \geq \frac{a}{c}, \quad (7)$$

$$\text{and} \quad \frac{b}{d} \geq 1 \geq \frac{a}{c}, \quad (8)$$

the capacity region of the Poisson interference channel is the convex hull of $\bigcup_{0 \leq p, q \leq 1} \mathcal{R}_{p,q}$ with $\mathcal{R}_{p,q}$ being:

$$\begin{aligned} R_1 &\leq \min \left\{ \mathbb{E}\psi(aA_1X_1^b, bA_2X_2^b; aA_1p, \lambda_1), \mathbb{E}\psi(cA_1X_1^b, dA_2X_2^b; cA_1p, \lambda_2) \right\}, \\ R_2 &\leq \min \left\{ \mathbb{E}\psi(bA_2X_2^b, aA_1X_1^b; bA_2q, \lambda_1), \mathbb{E}\psi(dA_2X_2^b, cA_1X_1^b; dA_2q, \lambda_2) \right\}, \\ R_1 + R_2 &\leq \min \left\{ \mathbb{E}\psi(aA_1X_1^b + bA_2X_2^b, 0; aA_1p + bA_2q, \lambda_1), \right. \\ &\quad \left. \mathbb{E}\psi(cA_1X_1^b + dA_2X_2^b, 0; cA_1p + dA_2q, \lambda_2) \right\}, \end{aligned} \quad (9)$$

in which the expectation is computed for binary input distributions with $P[X_1^b = 1] = 1 - P[X_1^b = 0] = p$ and $P[X_2^b = 1] = 1 - P[X_2^b = 0] = q$.

Proof: As discussed above, it suffices to show that, if the conditions in the theorem are

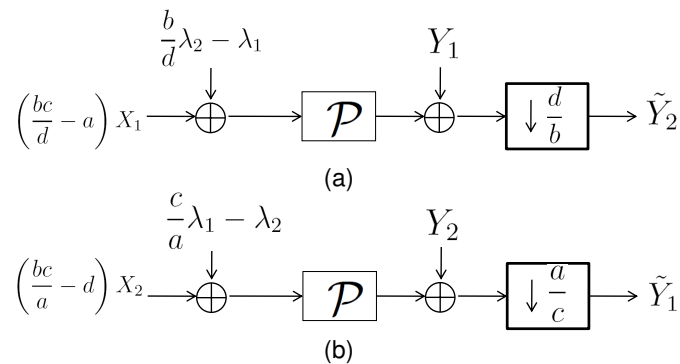


Fig. 3: The channel transformation at receivers

satisfied, any rate pair achievable in the Poisson interference channel is also achievable in the

Poisson compound MAC. We show this by certain enhancement and thinning arguments. Figure 3 (a) shows the operations involved at receiver 1. Suppose that a rate pair (R_1, R_2) is achievable for the Poisson interference channel. Hence, there exists a code $(T, \mathcal{M}_1, \mathcal{M}_2, \mu_1, \mu_2)$, such that $\frac{1}{T} \log |\mathcal{M}_i| \geq R_i - \epsilon$ for any $\epsilon > 0$ and for sufficiently large T . Furthermore, decoder 1 can decode $X_1(0, T)$ with an average error probability less than μ_1 . Once $X_1(0, T)$ is successfully decoded, decoder 1 generates a doubly stochastic Poisson process $\mathcal{P}((bc/d - a)X_1(0, T) + b\lambda_2/d - \lambda_1)$, which is independent of all other processes considered in the problem given $X_1(0, T)$, with the instantaneous rate $(bc/d - a)X_1(t) + b\lambda_2/d - \lambda_1$. We can do this as long as $bc/d - a \geq 0$ and $b\lambda_2/d - \lambda_1 \geq 0$. Decoder 1 then adds this process to Y_1 and obtain

$$\tilde{Y}_1 = Y_1 + \mathcal{P}((bc/d - a)X_1(0, T) + b\lambda_2/d - \lambda_1). \quad (10)$$

Due to the properties of the Poisson process [16], \tilde{Y}_1 is a doubly stochastic Poisson process with the instantaneous rate $bcX_1(t)/d + bX_2(t) + b\lambda_2/d$. Finally, decoder 1 constructs a process $\tilde{Y}_2 = \tilde{Y}_1 \downarrow (d/b)$. This can be done if $d/b \leq 1$. Again due to the properties of the Poisson process [16], \tilde{Y}_2 is a doubly stochastic Poisson process with the instantaneous rate

$$d/b(bcX_1(t)/d + bX_2(t) + b\lambda_2/d) = cX_1(t) + dX_2(t) + \lambda_2.$$

Hence, \tilde{Y}_2 has the same statistics as Y_2 . Since decoder 2 is able to decode $X_2(0, T)$ based on Y_2 with an error probability less than μ_2 , decoder 1 is able to decode $X_2(0, T)$ based on \tilde{Y}_2 with an error probability less than μ_2 as well, as long as it decodes X_1 successfully. Hence, receiver 1 is able to decode X_2 based on Y_1 with an error probability less than $\mu_1 + \mu_2$. To carry out the operations above, we require that

$$\frac{bc}{d} \geq a, \quad \frac{b\lambda_2}{d} - \lambda_1 \geq 0, \quad \frac{d}{b} \leq 1. \quad (11)$$

Similarly, Figure 3 (b) illustrates the operations involved at receiver 2. If the following conditions are satisfied, decoder 2 is able to decode both X_1 with an error probability less than $\mu_1 + \mu_2$ and X_2 with an error probability less than μ_2 based on Y_2 :

$$\frac{bc}{a} \geq d, \quad \frac{c\lambda_1}{a} - \lambda_2 \geq 0, \quad \frac{a}{c} \leq 1. \quad (12)$$

It is clear that conditions (11) and (12) are equivalent to (7) and (8). Hence, if these conditions are satisfied, the capacity region of the Poisson interference channel is the same as that of the compound MAC. The Poisson MAC has been studied in [6], which has shown that the Poisson

MAC can be approximated using binary input binary output MAC while reserving the capacity region. We use X_i^b to denote the input to the binary MAC obtained by this approximation. Using the results in [6], the capacity region of a compound MAC is given by the (9). ■

Remark 3.2: For the Gaussian interference channel, if the cross-link coefficients are large enough to fall into the so called very strong interference range, each transmitter can transmit at a rate as if there is no interference. Each receiver can first decode the other transmitter's message and then completely cancel the interference caused by the other user's signal. However, for the Poisson interference channel, this is not the case anymore. If $R_2 > 0$, then R_1 is strictly less than the capacity of the point-to-point channel from transmitter 1 to receiver 1 without interference. To see this, for any input distributions on X_1 and X_2 , we have

$$\begin{aligned}
 R_1 &\leq I(X_1(0, T); \mathcal{P}(aX_1(0, T) + bX_2(0, T) + \lambda_1) | bX_2(0, T)) \\
 &\leq \frac{1}{T} \int_0^T dt \mathbb{E} \psi(aX_1(t), bX_2(t); am_1(t), \lambda_1) \\
 &< \frac{1}{T} \int_0^T dt \mathbb{E} \psi(aX_1(t), 0; am_1(t), \lambda_1) \\
 &\leq C_{ni},
 \end{aligned} \tag{13}$$

in which the function $\psi(\cdot)$ is defined in (4), $m_1(t) = \mathbb{E}\{X_1(t)\}$, (13) is due to the fact that $R_2 > 0$ (and hence $X_2(t)$ is not always equal to 0) and the fact that $\psi(\cdot)$ is a decreasing function of β , and C_{ni} denotes the capacity of the point-to-point Poisson channel from transmitter 1 to receiver 1 without interference (i.e., the channel $X_1(0, T) \rightarrow \mathcal{P}(aX_1(0, T) + \lambda_1)$). Hence, for the Poisson interference channel, there is no corresponding very strong interference regime as that of the Gaussian interference channel.

IV. INNER BOUND FOR GENERAL POISSON INTERFERENCE CHANNELS

In this section, we derive an inner bound on the capacity region for general Poisson interference channels that do not satisfy the strong interference conditions (7) and (8). Following the approach introduced by Wyner [2], we first convert the Poisson interference channel into a binary discrete-time interference channel. We then obtain an achievable region for the binary discrete-time interference channel based on the Han-Kobayashi region [15] for certain input distributions.

Similarly to [2], we approximate the poisson interference channel by a binary input binary output (BIBO) discrete memoryless channel as follows. We fix some small constant $\Delta > 0$, and divide the transmission time into intervals with each having duration Δ . We further require that

- The transmitted signal $X_i(t)$ takes only values 0 and A_i , and is constant over the interval $(k\Delta, (k+1)\Delta]$, for every $k = 1, 2, \dots$;
- Receiver i observes only samples $Y_i(k\Delta)$, $k = 1, 2, \dots$; or alternatively, the increments $\hat{Y}_{i,k} = Y_i(k\Delta) - Y_i((k-1)\Delta)$. Furthermore, receiver i interprets $\hat{Y}_{i,k} \geq 2$ (a rare event when Δ is small) as being the same as $\hat{Y}_{i,k} = 0$. Thus receiver i obtains an output

$$Y_{i,k} = \begin{cases} 1, & \hat{Y}_{i,k} = 1 \\ 0, & \hat{Y}_{i,k} \neq 1 \end{cases}. \quad (14)$$

Although the capacity regions of both the general Poisson interference channel and the binary interference channel are unknown, we have the following result that shows that this approximation does not reduce the capacity region of the Poisson interference channel.

Theorem 4.1: Let $\{X_{i,m_i}(\cdot)\}_1^{M_i}$, $i = 1, 2$ be any code with parameters $(T, M_1, M_2, \mu_1, \mu_2)$ for the Poisson interference channel. Let $\epsilon > 0$ be arbitrary. Then there must exist a code $\{\hat{X}_{i,m_i}(\cdot)\}_1^{M_i}$ with parameters $(T, M_1, M_2, \hat{\mu}_1, \hat{\mu}_2)$ that satisfy $\hat{\mu}_i \leq \mu_i + \epsilon$ and the properties that for some N and $\Delta = T/N$,

- a) the decoder mapping $\hat{D}_i : \{Y_i(n\Delta)\}_{n=1}^N \rightarrow \{1, 2, \dots, M_i\}$;
- b) for each m_i , the waveform $\hat{X}_{i,m_i}(t)$ is constant on the interval $[(n-1)\Delta, n\Delta)$, $1 \leq n \leq N$; and $\hat{X}_{i,m_i}(t)$ takes only the values 0 or A_i ;
- c) decoder i declares an error whenever for some $n \in [1, N]$, $Y_i(n\Delta) - Y_i((n-1)\Delta) > 1$.

Proof: The proof adapts Wyner's proof for the optimality of binary approximation for the point-to-point Poisson channel [2] to take into account of the multiuser nature of the interference channel. Details can be found in Appendix A. ■

With this approximation, the Poisson interference channel can be converted into a binary discrete-time memoryless channel. In the following, we use X_i^b and Y_i^b , $i = 1, 2$ to denote the inputs and outputs of the corresponding binary interference channel, respectively. Based on Theorem 4.1, the transition probabilities $V_1(Y_1^b | X_1^b, X_2^b)$ (to receiver 1) and $V_2(Y_2^b | X_1^b, X_2^b)$ (to

receiver 2) of the binary interference channel are given by:

$$V_1(1|0, 0) = 1 - V_1(0|0, 0) = 1 - \exp^{-\lambda_1\Delta} = \lambda_1\Delta + O(\Delta^2), \quad (15)$$

$$V_1(1|1, 0) = 1 - V_1(0|1, 0) = 1 - \exp^{-(aA_1+\lambda_1)\Delta} = (aA_1 + \lambda_1)\Delta + O(\Delta^2), \quad (16)$$

$$V_1(1|0, 1) = 1 - V_1(0|0, 1) = 1 - \exp^{-(bA_2+\lambda_1)\Delta} = (bA_2 + \lambda_1)\Delta + O(\Delta^2), \quad (17)$$

$$V_1(1|1, 1) = 1 - V_1(0|1, 1) = 1 - \exp^{-(aA_1+bA_2+\lambda_1)\Delta} = (aA_1 + bA_2 + \lambda_1)\Delta + O(\Delta^2), \quad (18)$$

$$V_2(1|0, 0) = 1 - V_2(0|0, 0) = 1 - \exp^{-\lambda_2\Delta} = \lambda_2\Delta + O(\Delta^2), \quad (19)$$

$$V_2(1|1, 0) = 1 - V_2(0|1, 0) = 1 - \exp^{-(cA_1+\lambda_2)\Delta} = (cA_1 + \lambda_2)\Delta + O(\Delta^2), \quad (20)$$

$$V_2(1|0, 1) = 1 - V_2(0|0, 1) = 1 - \exp^{-(dA_2+\lambda_2)\Delta} = (dA_2 + \lambda_2)\Delta + O(\Delta^2), \quad (21)$$

$$V_2(1|1, 1) = 1 - V_2(0|1, 1) = 1 - \exp^{-(cA_1+dA_2+\lambda_2)\Delta} = (cA_1 + dA_2 + \lambda_2)\Delta + O(\Delta^2). \quad (22)$$

We then evaluate the simplified Han-Kobayashi region [17] of the binary interference channel using $|\mathcal{Q}| = 1$ (i.e., no time-sharing), W_1 and W_2 as binary random variables with $P_{W_1}(1) = 1 - P_{W_1}(0) = p_1$ and $P_{W_2}(1) = 1 - P_{W_2}(0) = p_2$. We further assume that X_1^b is generated from W_1 via a testing binary symmetric channel with parameter q_1 , and X_2^b is generated from W_2 via a testing binary symmetric channel with parameter q_2 . Using this input distribution, we can compute various mutual information terms involved in the Han-Kobayashi region. To obtain the achievable rate region of the original Poisson interference channel from the achievable region from this binary interference channel, we then need a proper normalization $1/\Delta$. The computation of various mutual information term using this input distribution is straightforward and hence is omitted for brevity.

V. OUTER BOUNDS FOR GENERAL POISSON INTERFERENCE CHANNELS

In the section, we derive several outer bounds on the capacity region for the Poisson interference channel when one or both of the strong interference conditions (7) and (8) are not satisfied.

A. Outer Bound for Various Interference Regimes

In this subsection, we derive several outer bounds with each being applicable to a certain parameter regime.

1) *The first outer bound is derived to be applicable when (8) is satisfied, but (7) is not satisfied.*

In this case, we have either $\lambda_1/\lambda_2 > b/d$ or $a/c > \lambda_1/\lambda_2$. We consider the case $\lambda_1/\lambda_2 > b/d$, the other case is similar. We derive an outer bound based on reducing the intensity of the noise. Consider an enhanced Poisson interference channel with output process at receiver 1 being $\tilde{Y}_1 = \mathcal{P}(aX_1(0, T) + bX_2(0, T) + \frac{b}{d}\lambda_2)$ and the output process at receiver 2 being the same as Y_2 . Since $\lambda_1/\lambda_2 > b/d$, we have $\frac{b}{d}\lambda_2 < \lambda_1$. It is clear that any rate pair (R_1, R_2) achievable for the original Poisson interference channel is also achievable for the channel specified by $(X_1, X_2) \rightarrow (\tilde{Y}_1, Y_2)$, because one can generate (\hat{Y}_1, Y_2) that has the same statistics as that of (Y_1, Y_2) by adding \tilde{Y}_1 with a randomly generated process $\mathcal{P}(\lambda_1 - \frac{b}{d}\lambda_2)$. Now, the channel $(X_1, X_2) \rightarrow (\tilde{Y}_1, Y_2)$ is a strong interference channel, since the condition in (7) is satisfied with the reduced noise. As the result, we have the following proposition.

Proposition 5.1: When (8) is satisfied and $\lambda_1/\lambda_2 > b/d$, the capacity region of the Poisson interference channel is inside of the convex hull of $\bigcup_{0 \leq p, q \leq 1} \mathcal{R}_{p, q}$ with $\mathcal{R}_{p, q}$ being:

$$\begin{aligned} R_1 &\leq \min \left\{ \mathbb{E}\psi \left(aA_1X_1^b, bA_2X_2^b; aA_1p, b\lambda_2/d \right), \mathbb{E}\psi \left(cA_1X_1^b, dA_2X_2^b; cA_1p, \lambda_2 \right) \right\}, \\ R_2 &\leq \min \left\{ \mathbb{E}\psi \left(bA_2X_2^b, aA_1X_1^b; bA_2q, b\lambda_2/d \right), \mathbb{E}\psi \left(dA_2X_2^b, cA_1X_1^b; dA_2q, \lambda_2 \right) \right\}, \\ R_1 + R_2 &\leq \min \left\{ \mathbb{E}\psi \left(aA_1X_1^b + bA_2X_2^b, 0; aA_1p + bA_2q, b\lambda_2/d \right), \right. \\ &\quad \left. \mathbb{E}\psi \left(cA_1X_1^b + dA_2X_2^b, 0; cA_1p + dA_2q, \lambda_2 \right) \right\}, \end{aligned} \quad (23)$$

with the expectation being computed for binary input distributions with $P[X_1^b = 1] = 1 - P[X_1^b = 0] = p$ and $P[X_2^b = 1] = 1 - P[X_2^b = 0] = q$.

2) *The second outer bound is applicable when (7) is satisfied but (8) is not satisfied.*

In this case, we have either $1 > b/d \geq a/c$ or $b/d \geq a/c > 1$. We consider the case $1 > b/d \geq a/c$, and the other case is similar. We derive an outer bound using arguments similar to that of Theorem 3.1. If the condition (7) and $1 > b/d \geq a/c$ are satisfied, we have that all operations in Figure 3 (b) are still valid. And hence, under these conditions, receiver 2 is able to decode both X_1 and X_2 . As the result, the capacity region of the Poisson interference channel remains the same if we require receiver 2 decode both messages. Hence, we have the following result.

Proposition 5.2: When (7) is satisfied and $1 > b/d$, the capacity region of the original Poisson

interference channel is inside of the convex hull of $\bigcup_{0 \leq p, q \leq 1} \mathcal{R}_{p,q}$ with $\mathcal{R}_{p,q}$ being:

$$\begin{aligned} R_1 &\leq \min \{ \mathbb{E}\psi(aA_1X_1^b, 0; aA_1p, \lambda_1), \mathbb{E}\psi(cA_1X_1^b, dA_2X_2^b; cA_1p, \lambda_2) \}, \\ R_2 &\leq \mathbb{E}\psi(dA_2X_2^b, cA_1X_1^b; dA_2q, \lambda_2), \\ R_1 + R_2 &\leq \mathbb{E}\psi(cA_1X_1^b + dA_2X_2^b, 0; cA_1p + dA_2q, \lambda_2), \end{aligned} \quad (24)$$

with the expectation being computed for binary input distributions with $P[X_1^b = 1] = 1 - P[X_1^b = 0] = p$ and $P[X_2^b = 1] = 1 - P[X_2^b = 0] = q$.

3) *The third outer bound is applicable when $a/c > b/d$.*

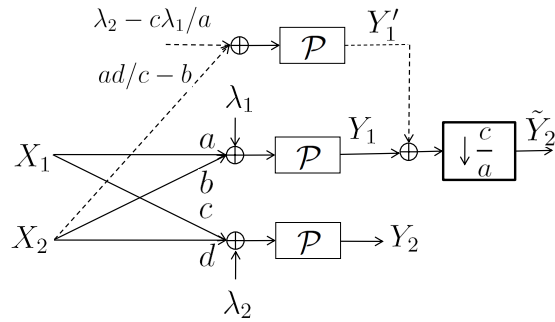


Fig. 4: The channel transformation at receiver 1

We derive an outer bound based on a genie aided argument. We first consider the situation when $a/c > 1$ and $a/c \geq \lambda_1/\lambda_2$. We derive this bound by providing extra information to receiver 1. Suppose a genie generates a doubly stochastic Poisson process $Y_1' = \mathcal{P}((ad/c-b)X_2(0, T) + (\lambda_2 - c\lambda_1/a))$, which is independent of other processes given $X_2(0, T)$, and provides this information to receiver 1 (See Figure 4 for an illustration.). Now, based on Y_1 and this additional information, receiver 1 can generate a Poisson process

$$\tilde{Y}_2 = (Y_1 + Y_1') \downarrow \frac{c}{a}.$$

It is easy to verify that the process \tilde{Y}_2 is a doubly stochastic Poisson process with instantaneous rate $cX_1(t) + dX_2(t) + \lambda_2$ and hence it has the same statistics as Y_2 . Since receiver 2 can decode X_2 from Y_2 with an error probability less than μ_2 , receiver 1 is also able to decode X_2 from \tilde{Y}_2 . As the result, the capacity region of the original Poisson interference channel should be inside of the capacity region of the new MAC with receiver 1 having two observation processes (Y_1, Y_1') .

We first have the following lemma.

Lemma 5.3: The capacity region of the MAC $(X_1, X_2) \rightarrow (Y_1, Y_1')$ is the convex hull of

$\bigcup_{0 \leq p, q \leq 1} \mathcal{R}_{p, q}$ with $\mathcal{R}_{p, q}$ being:

$$\begin{aligned} R_1 &\leq \mathbb{E}\psi(aA_1X_1^b, bA_2X_2^b; aA_1p, \lambda_1) \\ R_2 &\leq \mathbb{E}\psi(bA_2X_2^b, aA_1X_1^b; bA_2q, \lambda_1) + \mathbb{E}\psi((ad/c - b)A_2X_2^b, 0; (ad/c - b)A_2q, \lambda_2 - c\lambda_1/a), \\ R_1 + R_2 &\leq \mathbb{E}\psi(aA_1X_1^b + bA_2X_2^b, 0; aA_1p + bA_2q, \lambda_1) \\ &\quad + \mathbb{E}\psi((ad/c - b)A_2X_2^b, 0; (ad/c - b)A_2q, \lambda_2 - c\lambda_1/a), \end{aligned} \quad (25)$$

with the expectation being computed for binary input distributions with $P[X_1^b = 1] = 1 - P[X_1^b = 0] = p$ and $P[X_2^b = 1] = 1 - P[X_2^b = 0] = q$.

Proof: The proof follows closely from that of [6] and [18], and details are provided in Appendix B. ■

Based on the above lemma, we have the following proposition.

Proposition 5.4: When $a/c > 1$ and $a/c \geq \lambda_1/\lambda_2$, the capacity region of the Poisson interference channel is inside of the convex hull of $\bigcup_{0 \leq p, q \leq 1} \mathcal{R}_{p, q}$ with $\mathcal{R}_{p, q}$ being:

$$\begin{aligned} R_1 &\leq \mathbb{E}\psi(aA_1X_1^b, bA_2X_2^b; aA_1p, \lambda_1) \\ R_2 &\leq \min \left\{ \mathbb{E}\psi(dA_2X_2^b, 0; dA_2q, \lambda_2), \right. \\ &\quad \left. \mathbb{E}\psi(bA_2X_2^b, aA_1X_1^b; bA_2q, \lambda_1) + \mathbb{E}\psi((ad/c - b)A_2X_2^b, 0; (ad/c - b)A_2q, \lambda_2 - c\lambda_1/a) \right\} \\ R_1 + R_2 &\leq \mathbb{E}\psi(aA_1X_1^b + bA_2X_2^b, 0; aA_1p + bA_2q, \lambda_1) \\ &\quad + \mathbb{E}\psi((ad/c - b)A_2X_2^b, 0; (ad/c - b)A_2q, \lambda_2 - c\lambda_1/a), \end{aligned} \quad (26)$$

with the expectation being computed for binary input distributions with $P[X_1^b = 1] = 1 - P[X_1^b = 0] = p$ and $P[X_2^b = 1] = 1 - P[X_2^b = 0] = q$.

Proof: As $a/c > 1$ and $a/c \geq \lambda_1/\lambda_2$, all bounds in Lemma 5.3 hold. In addition,

$$R_2 \leq \mathbb{E}\psi(dA_2X_2^b, 0; dA_2q, \lambda_2), \quad (27)$$

as R_2 has to be smaller than the capacity of the interference free point-to-point channel from transmitter 2 to receiver 2, i.e., the channel $X_2 \rightarrow \mathcal{P}(dX_2 + \lambda_2)$. ■

For the case when $1 \geq a/c > b/d$, i.e., $d/b > c/a \geq 1$, we reverse the roles of receiver 1 and receiver 2 in the argument above. Namely, a genie provides receiver 2 with extra information

for it to decode X_1 . Then the capacity region of the original Poisson interference channel is contained in the capacity region of the enhanced MAC.

B. Genie-Aided Outer Bound for General Channels

The genie aided outer bound developed in Section V-A can be extended to be applicable for any parameter configurations as detailed in the following. Suppose a genie provides a doubly stochastic Poisson process $\hat{Y}_1 = \mathcal{P}(\delta_1 X_2(0, T) + \delta_2)$ to receiver 1. Here, given $X_2(0, T)$, \hat{Y}_1 is independent of all other processes of interest to this problem. Obviously, this does not decrease the capacity region of the Poisson interference channel. Since receiver 1 can decode $X_1(0, T)$ with an error probability less than μ_1 from the process Y_1 , receiver 1 can also generate a process $\tilde{Y}_1 = \mathcal{P}(\delta_3 X_1(0, T))$ that is independent of all other processes given $X_1(0, T)$. Now, we construct a process \hat{Y}_2 by setting

$$\hat{Y}_2 = (Y_1 + \hat{Y}_1 + \tilde{Y}_1) \downarrow \frac{1}{\delta}. \quad (28)$$

Then \hat{Y}_2 is a doubly stochastic Poisson process with instantaneous rate

$$\frac{a + \delta_3}{\delta} X_1(t) + \frac{b + \delta_1}{\delta} X_2(t) + \frac{\lambda_1 + \delta_2}{\delta}. \quad (29)$$

Now if we set

$$\begin{aligned} \delta_3 &= c\delta - a, \\ \delta_1 &= d\delta - b, \\ \delta_2 &= \lambda_2\delta - \lambda_1, \end{aligned} \quad (30)$$

then the process \hat{Y}_2 has the same statistics as that of Y_2 . Hence, from \hat{Y}_2 , receiver 1 is able to decode X_2 with an error probability less than μ_2 . To carry out the operations above, we require that

$$\delta \geq \max \left\{ 1, \frac{a}{c}, \frac{b}{d}, \frac{\lambda_1}{\lambda_2} \right\}. \quad (31)$$

As the result, receiver 1 should be able to decode both X_1 and X_2 from (Y_1, \hat{Y}_1) . Using the result in Lemma 5.3, we have the following bound for the sum rate:

$$\begin{aligned} R_1 + R_2 \leq \max_{0 \leq p, q \leq 1} \min_{\delta \geq \max \left\{ 1, \frac{a}{c}, \frac{b}{d}, \frac{\lambda_1}{\lambda_2} \right\}} \left\{ \mathbb{E} \psi(aA_1 X_1^b + bA_2 X_2^b, 0; aA_1 p + bA_2 q, \lambda_1) \right. \\ \left. + \mathbb{E} \psi((d\delta - b)A_2 X_2^b, 0; (d\delta - b)A_2 q, (\lambda_2\delta - \lambda_1)) \right\}, \end{aligned} \quad (32)$$

with the expectation being computed for binary input distributions with $P[X_1^b = 1] = 1 - P[X_1^b = 0] = p$ and $P[X_2^b = 1] = 1 - P[X_2^b = 0] = q$.

Following the steps similar to the above, we can provide genie-aided information to receiver 2 and derive the following bound on the sum rate

$$R_1 + R_2 \leq \max_{0 \leq p, q \leq 1} \min_{\gamma \geq \max\{1, \frac{c}{a}, \frac{d}{b}, \frac{\lambda_2}{\lambda_1}\}} \left\{ \mathbb{E}\psi(cA_1X_1^b + dA_2X_2^b, 0; cA_1p + dA_2q, \lambda_2) \right. \\ \left. + \mathbb{E}\psi((b\gamma - d)A_1X_1^b, 0; (b\gamma - d)A_1p, (\lambda_1\gamma - \lambda_2)) \right\}, \quad (33)$$

with the expectation being computed for binary input distributions with $P[X_1^b = 1] = 1 - P[X_1^b = 0] = p$ and $P[X_2^b = 1] = 1 - P[X_2^b = 0] = q$.

Proposition 5.5: The sum rate bounds in (32) and (33) combined with (23), (24) and (26) respectively serve as outer bounds on the capacity region of the Poisson interference channel.

C. Bounds Based on the Degraded Interference Channel

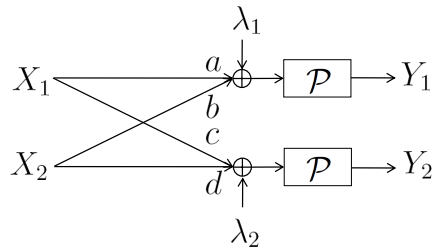
In this subsection, we derive an outer bound by constructing a degraded interference channel. The approach has the same flavor as that of the construction in the Gaussian interference channel [19].

We construct a new Poisson interference channel with the output process at decoder 1 being $\hat{Y}_1 = \mathcal{P}(aX_1(0, T) + \lambda_1)$ and the output process at decoder 2 being the same as Y_2 , as shown in Figure 5 (b). We use \mathcal{C}_1 to denote the capacity region of the original Poisson interference channel, i.e., the channel specified by $(X_1, X_2) \rightarrow (Y_1, Y_2)$. And use \mathcal{C}_2 to denote the capacity region of the interference channel specified by $(X_1, X_2) \rightarrow (\hat{Y}_1, Y_2)$. Obviously, $\mathcal{C}_1 \subseteq \mathcal{C}_2$. With \hat{Y}_1 , we can construct another interference channel $(X_1, X_2) \rightarrow (\hat{Y}_1, \hat{Y}_2)$ with the output processes \hat{Y}_1 at receiver 1 and \hat{Y}_2 at receiver 2 given by

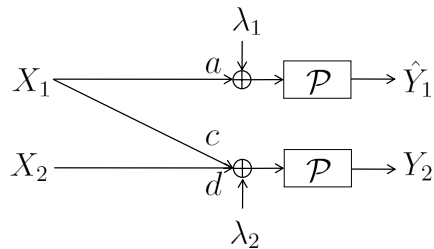
$$\hat{Y}_2 = \hat{Y}_1 \downarrow \frac{c}{a} + \mathcal{P}\left(dX_2(0, T) + \lambda_2 - \frac{c}{a}\lambda_1\right). \quad (34)$$

This is illustrated in Figure 5 (c). We use \mathcal{C}_3 to denote the capacity region of this channel. It is easy to see that \hat{Y}_2 has the same statistics as that of Y_2 , and hence $\mathcal{C}_2 = \mathcal{C}_3$. We construct the fourth Poisson interference channel with the output processes $(\tilde{Y}_1, \tilde{Y}_2)$ being

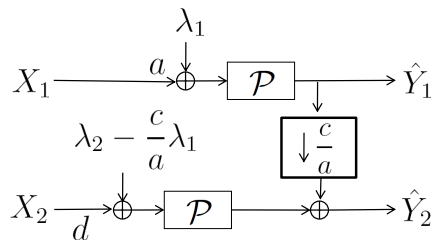
$$\tilde{Y}_1 = \mathcal{P}\left(a^*X_1(0, T) + \frac{a^*d}{c}X_2(0, T) + \lambda_1\right), \\ \tilde{Y}_2 = \tilde{Y}_1 \downarrow \frac{c}{a^*} + \mathcal{P}\left(\lambda_2 - \frac{c}{a^*}\lambda_1\right). \quad (35)$$



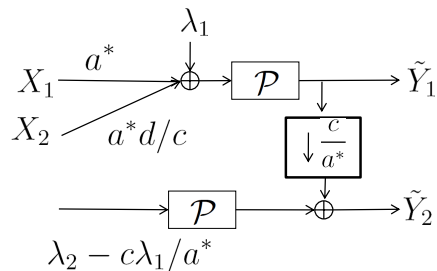
(a): Original channel



(b): Step 1



(c): Step 2



(d): Step 3

Fig. 5: Degraded Poisson broadcast channel conversion

This is illustrated in Figure 5 (d). We use \mathcal{C}_4 to denote the capacity region of this channel. The following lemma shows that we can find a constant a^* such that $\mathcal{C}_3 \subseteq \mathcal{C}_4$. Now the channel $(X_1, X_2) \rightarrow (\tilde{Y}_1, \tilde{Y}_2)$ is a degraded interference channel that satisfies $(X_1, X_2) \rightarrow \tilde{Y}_1 \rightarrow \tilde{Y}_2$.

Lemma 5.6: There exists a constant a^* such that $\mathcal{C}_3 \subseteq \mathcal{C}_4$.

Proof: Suppose $(R_1, R_2) \in \mathcal{C}_3$. It is easy to check that for any $a^* > c$, the statistics of \tilde{Y}_2 in Figure 5 (d) is the same as the statistics of \hat{Y}_2 in Figure 5 (c). Hence receiver 2 in Figure 5 (d) can decode X_2 with a small error probability. Furthermore, from Figure 5 (d), we have $(X_1, X_2) \rightarrow \tilde{Y}_1 \rightarrow \tilde{Y}_2$. Hence, receiver 1 in Figure 5 (d) should be able to decode X_2 from \tilde{Y}_1 as well.

Following Theorem 4.1, it is clear that limiting the input and output to be binary does not reduce the capacity region and the channel can be converted to a binary input binary output interference channel. Let $p = \Pr\{X_1^b = 1\}$, and $q = \Pr\{X_2^b = 1\}$. To simplify the notation, we denote

$$\begin{aligned} I_{X_1; \tilde{Y}_1 | X_2} &= \lim_{\Delta \rightarrow 0} \frac{1}{\Delta} I(X_1^b; \tilde{Y}_1^b | X_2^b), \\ I_{X_1; \hat{Y}_1} &= \lim_{\Delta \rightarrow 0} \frac{1}{\Delta} I(X_1^b; \hat{Y}_1^b). \end{aligned} \quad (36)$$

If $p = 0$ (or $p = 1$) or $q = 0$ (or $q = 1$), the problem degenerates to transmit W_1 or W_2 only. Hence, it suffices to limit the discussion to the case in which $0 < p < 1$ and $0 < q < 1$.

We will find the value of a^* in the following manner. First, for any given p and q , we find $a_{(p,q)}$ such that

$$I_{X_1; \tilde{Y}_1 | X_2} \geq I_{X_1; \hat{Y}_1}. \quad (37)$$

Then we set

$$a^* = \sup_{0 < p, q < 1} a_{(p,q)}. \quad (38)$$

In this way, receiver 1 in Figure 5 (d) can also decode X_1 , since by assumption receiver 1 in Figure 5 (c) can decode X_1 . As the result, $(R_1, R_2) \in \mathcal{C}_4$, which implies $(R_1, R_2) \in \mathcal{C}_3$. In the following we show that such a^* exists by finding $a_{(p,q)}$ that satisfies (37).

For a given p and q , we have

$$I_{X_1; \tilde{Y}_1 | X_2} = \bar{q}\phi(p, a_{(p,q)}A_1, \lambda_1) + q\phi(p, a_{(p,q)}A_1, \lambda_1 + a_{(p,q)}dA_2/c), \quad (39)$$

and

$$\begin{aligned} \frac{\partial I_{X_1; \tilde{Y}_1 | X_2}}{\partial a_{(p,q)}} &= \bar{q}pA_1[\ln(a_{(p,q)}A_1 + \lambda_1) - \ln(a_{(p,q)}A_1p + \lambda_1)] \\ &\quad + q[p(A_1 + dA_2/c) \ln(a_{(p,q)}(A_1 + dA_2/c) + \lambda_1) + \bar{p}dA_2/c \ln(a_{(p,q)}dA_2/c + \lambda_1) \\ &\quad - (A_1p + dA_2/c) \ln(a_{(p,q)}(A_1p + dA_2/c) + \lambda_1)]. \end{aligned} \quad (40)$$

We define $f(x) = x \ln(a_{(p,q)}x + \lambda_1)$, and it is easy to verify that $f(x)$ is strictly convex. Hence, $\mathbb{E}\{f(X)\} > f(\mathbb{E}\{X\})$, which implies

$$\begin{aligned} & p(A_1 + dA_2/c) \ln(a_{(p,q)}(A_1 + dA_2/c) + \lambda_1) + \bar{p}dA_2/c \ln(a_{(p,q)}dA_2/c + \lambda_1) \\ & - (A_1p + dA_2/c) \ln(a_{(p,q)}(A_1p + dA_2/c) + \lambda_1) > 0. \end{aligned} \quad (41)$$

Combining the above inequality with the fact that $\ln(a_{(p,q)}A_1 + \lambda_1) - \ln(a_{(p,q)}A_1p + \lambda_1) \geq 0$, we have

$$\frac{\partial I_{X_1; \tilde{Y}_1 | X_2}}{\partial a_{(p,q)}} > 0. \quad (42)$$

Furthermore, we have

$$\begin{aligned} \lim_{a_{(p,q)} \rightarrow \infty} \frac{\partial I_{X_1; \tilde{Y}_1 | X_2}}{\partial a_{(p,q)}} &= \bar{q}p \ln(1/p) + q[\bar{p}dA_2/c \ln(dA_2/c) \\ & + p(A_1 + dA_2/c) \ln(A_1 + dA_2/c) - (A_1p + dA_2/c) \ln(A_1p + dA_2/c)] > 0, \end{aligned} \quad (43)$$

which holds due to the fact that $\ln(1/p) \geq 0$ and that $g(x) = x \ln x$ is strictly convex.

The inequality (42) implies that $I_{X_1; \tilde{Y}_1 | X_2}$ is an increasing function of $a_{(p,q)}$. Furthermore, following from (43), $I_{X_1; \tilde{Y}_1 | X_2}$ does not saturate as $a_{(p,q)} \rightarrow \infty$. Hence, for any given p and q , we can find $a_{(p,q)}$ such that

$$I_{X_1; \tilde{Y}_1 | X_2} \geq I_{X_1; \hat{Y}_1}, \quad (44)$$

as $I_{X_1; \hat{Y}_1}$ is a constant for a given p . This concludes the proof. \blacksquare

Remark 5.7: If receiver 1 could have completely cancelled the interference by knowing X_2 , a^* could be chosen as $a^* = a$. However, as discussed above, after decoding X_2 , receiver 1 is not able to completely cancel the interference generated by X_2 . Hence, we need to increase the value a^* so that receiver 1 is able to decode X_1 .

Remark 5.8: From the steps above, it is clear that the construction works if

$$a > c, \lambda_2 > c\lambda_1/a. \quad (45)$$

In fact, the construction also works if

$$d > b, \lambda_1 > b\lambda_2/d, \quad (46)$$

by reversing the roles of receiver 1 and receiver 2.

The capacity region \mathcal{C}_4 of the degraded interference channel $(X_1, X_2) \rightarrow (\tilde{Y}_1, \tilde{Y}_2)$ in Figure 5 (d) is further contained by the capacity region of a degraded Poisson broadcast channel:

$$\tilde{Y}_1 = \mathcal{P}(X + \lambda_1), \quad (47)$$

$$\tilde{Y}_2 = \mathcal{P}(cX/a^* + \lambda_2), \quad (48)$$

in which

$$X = a^*X_1 + a^*dX_2/c \quad (49)$$

is the input of the broadcast channel. In this broadcast channel, the maximum value of the input is $a^*A_1 + a^*dA_2/c$. The capacity region of the degraded Poisson broadcast channel has been characterized in [7]. In particular, similar to Theorem 4.1, one can approximate the degraded Poisson broadcast channel as a binary broadcast channel. Furthermore, the binary broadcast channel is also degraded if $a^* > c$ and $\lambda_2 > c\lambda_1/a^*$. Hence, one can apply the capacity region result of the discrete degraded broadcast channel [20]. Let

$$\begin{aligned} I_{X;Y_1|U} &= \lim_{\Delta \rightarrow 0} \frac{1}{\Delta} I(X^b; Y_1^b|U), \\ I_{U;Y_2} &= \lim_{\Delta \rightarrow 0} \frac{1}{\Delta} I(U; Y_2^b). \end{aligned} \quad (50)$$

The capacity region of the degraded Poisson broadcast channel can be characterized as

$$\mathcal{C}_5 = \bigcup_{U, X^b} \{R_1 \leq I_{X;Y_1|U}, R_2 \leq I_{U;Y_2}\} \quad (51)$$

with $U \rightarrow X^b \rightarrow Y_1^b \rightarrow Y_2^b$ and $|\mathcal{U}| \leq \min\{\mathcal{Y}_1^b, \mathcal{Y}_2^b, \mathcal{X}^b\}$. In our case, we have $|\mathcal{U}| \leq 2$.

In summary, we have the following proposition.

Proposition 5.9: If $a > c$, $\lambda_2 > c\lambda_1/a$, the capacity region of the Poisson interference channel is contained in

$$\bigcup_{\alpha, p, q} \{R_1 \leq I_{X;Y_1|U}, R_2 \leq I_{U;Y_2}\}$$

with

$$\begin{aligned} I_{U;Y_2} &= -[\alpha(p_0\lambda_2 + \bar{p}_0(cA_1 + dA_2 + \lambda_2)) + \bar{\alpha}(q_0\lambda_2 + \bar{q}_0(cA_1 + dA_2 + \lambda_2))] \\ &\quad \ln[\alpha(p_0\lambda_2 + \bar{p}_0(cA_1 + dA_2 + \lambda_2)) + \bar{\alpha}(q_0\lambda_2 + \bar{q}_0(cA_1 + dA_2 + \lambda_2))] \\ &\quad + \alpha(p_0\lambda_2 + \bar{p}_0(cA_1 + dA_2 + \lambda_2)) \ln(p_0\lambda_2 + \bar{p}_0(cA_1 + dA_2 + \lambda_2)) \\ &\quad + \bar{\alpha}(q_0\lambda_2 + \bar{q}_0(cA_1 + dA_2 + \lambda_2)) \ln(q_0\lambda_2 + \bar{q}_0(cA_1 + dA_2 + \lambda_2)), \end{aligned} \quad (52)$$

and

$$\begin{aligned}
I_{X;Y_1|U} &= -\alpha[p_0\lambda_1 + \bar{p}_0(A_1a^* + A_2a^*d/c + \lambda_1)] \ln[p_0\lambda_1 + \bar{p}_0(A_1a^* + A_2a^*d/c + \lambda_1)] \\
&\quad -\bar{\alpha}_0[q_0\lambda_1 + \bar{q}_0(A_1a^* + A_2a^*d/c + \lambda_1)] \ln[q_0\lambda_1 + \bar{q}_0(A_1a^* + A_2a^*d/c + \lambda_1)] \\
&\quad +(\alpha p_0 + \bar{\alpha}_0 q_0)\lambda_1 \ln \lambda_1 + (\alpha \bar{p}_0 + \bar{\alpha}_0 \bar{q}_0)(A_1a^* + A_2a^*d/c + \lambda_1) \ln(A_1a^* + A_2a^*d/c + \lambda_1).
\end{aligned}$$

VI. POISSON Z-INTERFERENCE CHANNEL

In this section, we study the Poisson Z-interference channel illustrated in Figure 2. The sum capacity of the Gaussian Z-interference channel has been determined in [21] for all parameter ranges. The argument relies on the ability to completely cancel the interference once the signal is decoded in the Gaussian channel. However, such an argument does not work for the Poisson channel. In the following, we characterize the sum capacity of the Z-interference channel in certain parameter range by studying the corresponding discrete-time memoryless binary Z-interference channel and thus leveraging the capacity results for the discrete-time memoryless Z-interference channel [22].

We first determine the sum capacity of the Poisson Z-interference channel when c is sufficiently small, i.e., in the weak interference regime.

Theorem 6.1: For the Poisson Z-interference channel, if

$$c < \min \{a, a\lambda_2/\lambda_1\}, \quad (53)$$

then the sum capacity is given by

$$C_{sum} = \lim_{0 \leq p \leq 1} [\phi(p, aA_1, \lambda_1) + \frac{u + dA_2}{e} (1 + dA_2/u)^{u/(dA_2)} + u(1 + u/(dA_2)) \ln(1 + dA_2/u)], \quad (54)$$

with $u = pcA_1 + \lambda_2$. Furthermore, the sum capacity is achieved by receiver 2 treating the signal of transmitter 1 as noise.

Proof: First, following from Theorem 4.1, the capacity region of the Poisson Z-interference is the same as the capacity region of the following discrete-time memoryless binary Z-interference channel (after normalizing by $1/\Delta$). Again, we use X_i^b and Y_i^b , $i = 1, 2$ to denote the input and output of the corresponding binary Z-interference channel. Using the binary approximation discussed in Section IV, the transition probabilities $V_1(Y_1^b|X_1^b)$ (to receiver 1, as it is independent

of X_2^b) and $V_2(Y_2^b|X_1^b, X_2^b)$ (to receiver 2) of the binary interference channel are given by:

$$V_1(1|0) = 1 - V_1(0|0) = \lambda_1\Delta + O(\Delta^2), \quad (55)$$

$$V_1(1|1) = 1 - V_1(0|1) = (aA_1 + \lambda_1)\Delta + O(\Delta^2), \quad (56)$$

$$V_2(1|0, 0) = 1 - V_2(0|0, 0) = \lambda_2\Delta + O(\Delta^2), \quad (57)$$

$$V_2(1|0, 1) = 1 - V_2(0|0, 1) = (dA_2 + \lambda_2)\Delta + O(\Delta^2), \quad (58)$$

$$V_2(1|1, 0) = 1 - V_2(0|1, 0) = (cA_1 + \lambda_2)\Delta + O(\Delta^2), \quad (59)$$

$$V_2(1|1, 1) = 1 - V_2(0|1, 1) = (cA_1 + dA_2 + \lambda_2)\Delta + O(\Delta^2). \quad (60)$$

Using these transition probabilities, in the following, we show that if

$$c < \min \{a, a\lambda_2/\lambda_1\}, \quad (61)$$

then there exists a distribution $W(Y_2^b|X_2^b, Y_1^b)$, such that

$$V_2(y_2^b|x_1^b, x_2^b) = \sum_{y_1^b} V_1(y_1^b|x_1^b)W(y_2^b|x_2^b, y_1^b), \forall (x_1^b, x_2^b, y_2^b). \quad (62)$$

We can characterize $W(Y_2^b|X_2^b, Y_1^b)$ by the following parameters

$$q_1 = W(1|0, 0) = 1 - W(0|0, 0), \quad (63)$$

$$q_2 = W(1|1, 0) = 1 - W(0|1, 0), \quad (64)$$

$$q_3 = W(1|0, 1) = 1 - W(0|0, 1), \quad (65)$$

$$q_4 = W(1|1, 1) = 1 - W(0|1, 1). \quad (66)$$

The construction is illustrated in Figure 6 (in the figure, we show only the transition probabilities to “1”). To satisfy the conditions in (62), $q_i, i = 1, \dots, 4$ should be chosen such that (ignoring $O(\Delta^2)$ terms):

$$V_2(1|0, 0) = \lambda_2\Delta = q_1(1 - \lambda_1\Delta) + q_3\lambda_1\Delta, \quad (67)$$

$$V_2(1|0, 1) = (dA_2 + \lambda_2)\Delta = q_2(1 - \lambda_1\Delta) + q_4\lambda_1\Delta, \quad (68)$$

$$V_2(1|1, 0) = (cA_1 + \lambda_2)\Delta = q_1(1 - (aA_1 + \lambda_1)\Delta) + q_3(aA_1 + \lambda_1)\Delta, \quad (69)$$

$$V_2(1|1, 1) = (cA_1 + dA_2 + \lambda_2)\Delta = q_2(1 - (aA_1 + \lambda_1)\Delta) + q_4(aA_1 + \lambda_1)\Delta. \quad (70)$$

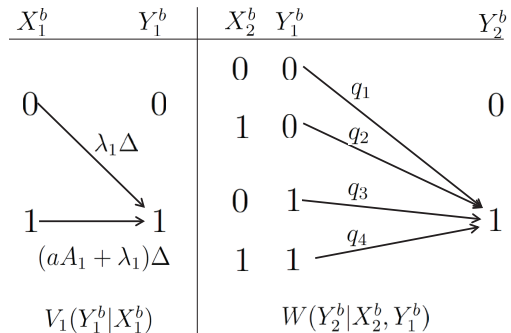


Fig. 6: Construction of a degraded Z-interference channel

From these equations, we obtain

$$q_1 = \frac{\Delta(a\lambda_2 - c\lambda_1)}{a}, \quad (71)$$

$$q_2 = \frac{\Delta(adA_2 + a\lambda_2 - c\lambda_1)}{a}, \quad (72)$$

$$q_3 = \frac{c + (a\lambda_2 - c\lambda_1)\Delta}{a}, \quad (73)$$

$$q_4 = \frac{c + \Delta(adA_2 + a\lambda_2 - c\lambda_1)}{a}. \quad (74)$$

Now, one can verify that, if (53) holds, then we have $0 < q_i < 1$ (we need strict inequalities to take care of $O(\Delta^2)$ terms), when Δ is sufficiently small.

Since the capacity region of the Z-interference channel depends only on the marginal distributions of two receivers, the capacity region of the channel specified in (55)- (60) is the same as the capacity region of the channel specified in Figure 6. The channel specified in Figure 6 satisfies $X_1^b \rightarrow (X_2^b, Y_1^b) \rightarrow Y_2^b$. The sum capacity of a discrete-time memoryless Z-interference channel that satisfies this Markov chain property has been determined in [22]. Applying the result in [22], we have the sum capacity of the Poisson Z-interference channel given by

$$C_{sum} = \lim_{\Delta \rightarrow 0} \frac{1}{\Delta} \max_{p,q} \{I(X_1^b; Y_1^b) + I(X_2^b; Y_2^b)\} = \max_{p,q} \{I_{X_1; Y_1} + I_{X_1; Y_2}\} \quad (75)$$

in which $p = \Pr\{X_1^b = 1\}$ and $q = \Pr\{X_2^b = 1\}$, and

$$I_{X_1; Y_1} \triangleq \lim_{\Delta \rightarrow 0} \frac{1}{\Delta} I(X_1^b; Y_1^b),$$

$$I_{X_1; Y_2} \triangleq \lim_{\Delta \rightarrow 0} \frac{1}{\Delta} I(X_2^b; Y_2^b).$$

This sum capacity is achieved by treating the other user's signal (if any) as noise.

It is easy to verify that (75) can be written as

$$C_{sum} = \lim_{0 \leq p, q \leq 1} [\phi(p, aA_1, \lambda_1) + \phi(q, dA_2, pcA_1 + \lambda_2)]. \quad (76)$$

To solve this optimization problem, we observe that $\phi(p, aA_1, \lambda_1)$ does not depend on q . For any p , by a simple calculation, it can be shown that the optimal value of q that maximizes $\phi(q, dA_2, pcA_1 + \lambda_2)$ is given by

$$q^* = \frac{u}{dA_2} \left[\frac{1}{e} \left(1 + \frac{dA_2}{u} \right)^{1 + \frac{u}{dA_2}} - 1 \right]. \quad (77)$$

After detailed calculation by plugging (77) in (76), we have

$$C_{sum} = \lim_{0 \leq p \leq 1} \left[\phi(p, aA_1, \lambda_1) + \frac{u + dA_2}{e} (1 + dA_2/u)^{u/(dA_2)} + u(1 + u/(dA_2)) \ln(1 + dA_2/u) \right], \quad (78)$$

which concludes the proof. ■

The condition in Theorem 6.1 can also be understood based on the transformation illustrated in Figure 7. From the figure, we can see that the capacity regions of these two channels are equivalent if the conditions in Theorem 6.1 are satisfied. Furthermore, we have $X_1 \rightarrow (X_2, Y_1) \rightarrow Y_2$ in the transformed channel.

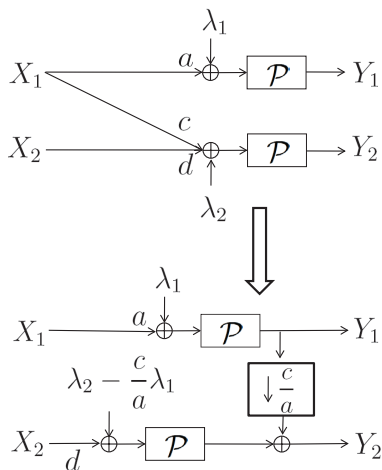


Fig. 7: The transformation for the Poisson Z-interference channel

In all numerical tests we have tried, the objective function in (78) is concave. However, we are not able to analytically show whether the function to be optimized is concave or not, because the form is too complicated.

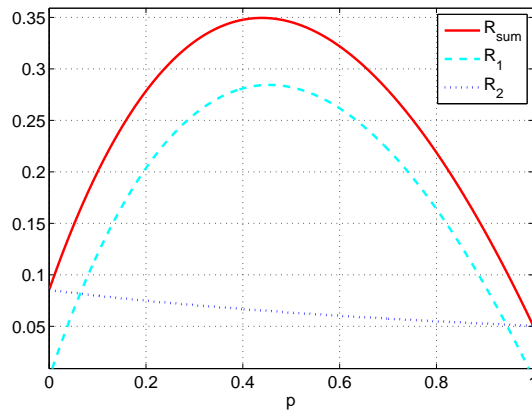


Fig. 8: The sum capacity for the Poisson Z-interference channel

For the Gaussian Z-interference channel with weak interference, in order to achieve the sum capacity, the transmitter whose corresponding receiver does not experience interference should transmit at the full possible rate [21]. The following example shows that this is not the case for the Poisson Z-interference channel anymore. In this example, we set $A_1 = A_2 = c = d = \lambda_1 = \lambda_2 = 1$ and $a = 2.1$. It is easy to verify that these choices of parameters satisfy the conditions in Theorem 6.1. Figure 8 plots the values of R_1 , R_2 and R_{sum} as functions of p . To maximize R_1 , i.e., to maximize the rate of the user not experiencing interference, we should choose $p = 0.455$. However, to maximize the sum rate, we should choose $p = 0.439$. Hence, for the Poisson Z-interference channel with weak interference, to achieve the sum capacity, the user not experiencing interference does not necessarily transmit at the maximum rate. It is also easy to show that the optimal value of p_{sum}^* that maximizes the sum rate is always less or equal to the optimal value of p_1^* that maximizes R_1 of user 1 only.

We next characterize the sum capacity when the interference is sufficiently strong, i.e., in the strong interference regime.

Theorem 6.2: Define c^* as the minimal value of c such that

$$\min_{0 \leq p \leq 1} \{ \phi(p, cA_1, dA_2 + \lambda_2) - \phi(p, aA_1, \lambda_1) \} = 0. \quad (79)$$

For the Poisson Z-interference channel, if $c \geq c^*$, the sum capacity is given by

$$C_{sum} = \max_{0 \leq p, q \leq 1} \{ \phi(p, aA_1, \lambda_1) + \bar{p}\phi(q, dA_2, \lambda_2) + p\phi(q, dA_2, cA_1 + \lambda_2) \}. \quad (80)$$

Remark 6.3: We note that in (80), $\phi(p, aA_1, \lambda_1)$ is not a function of q , while $\bar{p}\phi(q, dA_2, \lambda_2) + p\phi(q, dA_2, cA_1 + \lambda_2)$ is a concave function of q for a given p . Hence the optimization problem in (80) can be converted to a one-dimensional optimization problem by first solving the optimal value of q^* as a function of p that maximizes $\bar{p}\phi(q, dA_2, \lambda_2) + p\phi(q, dA_2, cA_1 + \lambda_2)$ and then optimizing the resulting objective function over p .

Proof: We consider the binary Z-interference channel obtained via the binary approximation. From Theorem 4.1, the capacity region of the Poisson Z-interference channel is the same as the binary Z-interference channel. For any $0 \leq p, q \leq 1$, we have

$$\begin{aligned} I_{X_1;Y_1} &:= \lim_{\Delta \rightarrow 0} \frac{1}{\Delta} I(X_1^b; Y_1^b) = \phi(p, aA_1, \lambda_1), \\ I_{X_1;Y_2} &:= \lim_{\Delta \rightarrow 0} \frac{1}{\Delta} I(X_1^b; Y_2^b) = \phi(p, cA_1, qdA_2 + \lambda_2), \\ I_{X_1;Y_2|X_2} &:= \lim_{\Delta \rightarrow 0} \frac{1}{\Delta} I(X_1^b; Y_2^b | X_2^b) = \bar{q}\phi(p, cA_1, \lambda_2) + q\phi(p, cA_1, dA_2 + \lambda_2), \\ I_{X_2;Y_2|X_1} &:= \lim_{\Delta \rightarrow 0} \frac{1}{\Delta} I(X_2^b; Y_2^b | X_1^b) = \bar{p}\phi(q, dA_2, \lambda_2) + p\phi(q, dA_2, cA_1 + \lambda_2). \end{aligned}$$

Converse:

We first have a simple outer bound on the capacity region. For any given $0 \leq p, q \leq 1$, define $\mathcal{R}_{p,q}$ be the set of (R_1, R_2) satisfying the following conditions:

$$R_1 \leq I_{X_1;Y_1}, \quad (81)$$

$$R_2 \leq I_{X_2;Y_2|X_1}, \quad (82)$$

It is easy to see that the convex hull of $\bigcup_{p,q} \mathcal{R}_{p,q}$ is an outer bound of the capacity region.

Achievability:

If we additionally require receiver 2 to decode messages from transmitter 1, then any rate achievable pair achievable with this additional requirement is also achievable for the Poisson Z-interference channel. For any given $0 \leq p, q \leq 1$, let $\mathcal{R}_{p,q}^a$ be the set of (R_1, R_2) satisfying the following conditions:

$$R_1 \leq I_{X_1;Y_1}, \quad (83)$$

$$R_1 \leq I_{X_1;Y_2|X_2}, \quad (84)$$

$$R_2 \leq I_{X_2;Y_2|X_1}, \quad (85)$$

$$R_1 + R_2 \leq I_{X_1, X_2; Y_2}. \quad (86)$$

It is clear that the convex hull of $\bigcup_{p,q} \mathcal{R}_{p,q}^a$ is an inner bound for the capacity region of the Poisson Z-interference channel.

Now, we simplify $\mathcal{R}_{p,q}^a$. First, we have

$$\frac{\partial \phi(p, A, \lambda)}{\partial \lambda} = \bar{p} \ln \lambda + p \ln(A + \lambda) - \ln(Ap + \lambda) \leq 0, \quad (87)$$

which implies that $\phi(p, A, \lambda)$ is a non-increasing function of λ . Hence

$$\phi(p, cA_1, \lambda_2) \geq \phi(p, cA_1, dA_2 + \lambda_2),$$

which implies that for any p , we have

$$\min_{0 \leq q \leq 1} I_{X_1; Y_2 | X_2} = \phi(p, cA_1, dA_2 + \lambda_2). \quad (88)$$

As the result, if

$$\min_{0 \leq p \leq 1} \{\phi(p, cA_1, dA_2 + \lambda_2) - \phi(p, aA_1, \lambda_1)\} = 0, \quad (89)$$

we have

$$I_{X_1; Y_2 | X_2} \geq I_{X_1; Y_1}, \quad \text{for all } 0 \leq p, q \leq 1, \quad (90)$$

which implies that (84) is redundant.

In addition, from (83) and (85), we have

$$\begin{aligned} R_1 + R_2 &\leq I_{X_1; Y_1} + I_{X_2; Y_2 | X_1} \\ &\stackrel{(a)}{\leq} I_{X_1; Y_2} + I_{X_2; Y_2 | X_1} \\ &= I_{X_1, X_2; Y_2}, \end{aligned} \quad (91)$$

where (a) follows because

$$I_{X_1; Y_2} = \phi(p, cA_1, qdA_2 + \lambda_2) \quad (92)$$

$$\stackrel{(b)}{\geq} \phi(p, cA_1, dA_2 + \lambda_2) \quad (93)$$

$$\stackrel{(c)}{\geq} \phi(p, aA_1, \lambda_1) \quad (94)$$

$$= I_{X_1; Y_1}, \quad (95)$$

in which (b) is true due to (87), (c) is true if (79) holds. This implies that (86) is redundant.

As the result, the region $\mathcal{R}_{p,q}^a$ is the same as

$$R_1 \leq I_{X_1;Y_1}, \quad (96)$$

$$R_2 \leq I_{X_2;Y_2|X_1}. \quad (97)$$

Since the simplified $\mathcal{R}_{p,q}^a$ is the same as $\mathcal{R}_{p,q}$, the sum rate (actually the entire region) can be achieved by the scheme that transmitter 1 transmits at a rate that both receiver 1 and receiver 2 can decode (when receiver 2 decodes, it treats signals from transmitter 1 as interference) the message from transmitter 1, then transmitter 2 sends at the rate $I_{X_2;Y_2|X_1}$ and receiver 2 decodes the message from transmitter 2 after first decoding the message from transmitter 1.

Hence, the sum rate capacity is given by

$$C_{sum} = \max_{0 \leq p, q \leq 1} \{ \phi(p, aA_1, \lambda_1) + \bar{p}\phi(q, dA_2, \lambda_2) + p\phi(q, dA_2, cA_1 + \lambda_2) \}. \quad (98)$$

■

Remark 6.4: The scenario is similar to that of the strong Poisson interference channel discussed in Theorem 3.1 in the sense that the cross link coefficient is sufficiently large. However, since $b = 0$, the construction in the proof of Theorem 3.1 does not work here anymore.

VII. CONCLUSIONS

We have studied the Poisson interference channel, a suitable model for multiuser optical communications. We have derived the conditions for the strong interference channel under which the capacity region remains the same even if we require both decoders to decode both messages. We have also derived several inner and outer bounds for the capacity region. We have also characterized the sum rate capacity for the Poisson Z-interference channel when the cross link coefficient is either sufficiently small or sufficiently large.

For the future work, it will be interesting to obtain finite gap results similar to that of the Gaussian interference channel [13]. It is also of interest to investigate band-limited channels, which have been studied in the point-to-point setup in [5], in this multi-user setup. Another direction to pursue is to exploit the relationship between the mutual information and conditional mean estimations in Poisson channels established in [14]. The relationship between mutual information and conditional mean estimations for the Gaussian channel [23] has provided important insights on the Gaussian Z-interference channels [24]. Although Poisson channels and Gaussian

channels are different in terms of neutralizing interference, the unifying framework proposed in [25] will be useful for this line of investigation.

APPENDIX A

PROOF OF THEOREM 4.1

The proof follows that of Theorem 2.1 in [2] with adaptations. In particular, the proof first follows the construction steps in [2] until (113). However, to set up the notations for the entire proof and for the completeness of the proof, we repeat them here with slightly different notations.

Let $\{X_{i,m_i}(\cdot)\}_1^{|\mathcal{M}_i|}$ be the code in the hypothesis of the theorem, and let $D_i : \mathcal{S}_i(T) \rightarrow \{1, \dots, |\mathcal{M}_i|\}$ be the corresponding decoding mapping. Here $\mathcal{S}_i(T)$ is the space of step functions $Y_i(0, T)$ observed by receiver i . Let $\epsilon > 0$ be given. For $1 \leq m_i \leq |\mathcal{M}_i|$, let B_{i,m_i} be the set of sample paths mapped by D_i to m_i , i.e.,

$$B_{i,m_i} = \{Y_i(\cdot) : D_i(Y_i) = m_i\}. \quad (99)$$

For any event B_i in $\mathcal{S}_i(T)$, and $1 \leq m_i \leq M_i$, we write

$$\Pr\{B_i | X_{i,m_i}(\cdot)\} = P_{m_i}(B_i). \quad (100)$$

Hence

$$\begin{aligned} 1 - \mu_i &= \frac{1}{|\mathcal{M}_i|} \sum_{m_i=1}^{|\mathcal{M}_i|} \Pr\{D_i(Y_i) = m_i | X_{i,m_i}(\cdot)\} \\ &= \frac{1}{|\mathcal{M}_i|} \sum_{m_i=1}^{|\mathcal{M}_i|} P_{m_i}(B_{i,m_i}). \end{aligned} \quad (101)$$

Let $\mathcal{A}_i(T)$ be the σ -field of events in $\mathcal{S}_i(T)$ generated by $\hat{\mathcal{A}}_i(T)$, which are the class of events defined by finite sets of samples of $Y_i(0, T)$, i.e., $(Y_i(t_1), \dots, Y_i(t_K))$, where $t_k \in [0, T]$ and $1 \leq k < K < \infty$. From the argument based on measure theory ([26], page 56), we have that for any $B_i \in \mathcal{A}_i(T)$ and arbitrary $\delta > 0$, there exists a $\tilde{B}_i \in \hat{\mathcal{A}}_i(T)$ such that the symmetric difference

$$P_{m_i}(B_i \ominus \tilde{B}_i) \leq \delta,$$

in which the symmetric difference is defined as

$$B_i \ominus \tilde{B}_i = (B_i \setminus \tilde{B}_i) \cup (\tilde{B}_i \setminus B_i).$$

It then follows that for a set of events $\{B_{i,m_i}\}$ and arbitrary $\epsilon > 0$, there exist a corresponding set of events $\{\tilde{B}_{i,m_i}\}$ defined by a suitably rich set of samples such that for all $m_i, m'_i \in [1, |\mathcal{M}_i|]$,

$$P_{m_i}(B_{i,m'_i} \ominus \tilde{B}_{i,m'_i}) \leq \epsilon/(2|\mathcal{M}_i|). \quad (102)$$

Corresponding to the events $\{\tilde{B}_{i,m_i}\}$, we define the disjoint sets $\{\hat{B}_{i,m_i}\}$ by

$$\hat{B}_{i,1} = \tilde{B}_{i,1} \quad (103)$$

$$\hat{B}_{i,m_i} = \tilde{B}_{i,m_i} - \bigcup_{m'_i < m_i} \hat{B}_{m'_i}, \quad 2 \leq m_i \leq |\mathcal{M}_i|. \quad (104)$$

The disjoint set of events $\{\hat{B}_{i,m_i}\}$ define the decoder \hat{D}_i , if $\hat{D}_i(Y_i(0, T)) = m_i$, when $Y_i(0, T) \in \hat{B}_{i,m_i}$. Furthermore, a finite set $\{t_k\}$ exists with

$$0 \leq t_1 \leq \dots \leq t_K \leq T$$

such that the \hat{B}_{i,m_i} are determined by the samples $\{Y_i(t_k)\}_{k=1}^K$. We then have

$$\begin{aligned} \hat{\mu}_i &= 1 - \frac{1}{|\mathcal{M}_i|} \sum_{m_i=1}^{|\mathcal{M}_i|} P_{m_i}(\hat{B}_{i,m_i}) \\ &\leq 1 - \frac{1}{|\mathcal{M}_i|} \sum_{m_i=1}^{|\mathcal{M}_i|} P_{m_i}(B_{i,m_i} \hat{B}_{i,m_i}) \\ &= 1 - \frac{1}{|\mathcal{M}_i|} \sum_{m_i=1}^{|\mathcal{M}_i|} \{P_{m_i}(B_{i,m_i}) - P_{m_i}(B_{i,m_i} \hat{B}_{i,m_i}^c)\} \\ &= \mu_i + \frac{1}{|\mathcal{M}_i|} \sum_{m_i=1}^{|\mathcal{M}_i|} P_{m_i}(B_{i,m_i} \hat{B}_{i,m_i}^c). \end{aligned} \quad (105)$$

It is easy to check that

$$\hat{B}_{i,m_i} = \tilde{B}_{i,m_i} - \bigcup_{m'_i < m_i} \hat{B}_{m'_i} = \tilde{B}_{i,m_i} - \bigcup_{m'_i < m_i} \tilde{B}_{m'_i} \quad (106)$$

$$= \tilde{B}_{i,m_i} \bigcap_{m'_i < m_i} \tilde{B}_{i,m'_i}^c. \quad (107)$$

Thus using (102), (107) and following [2], we have $P_{m_i}(B_{i,m_i} \hat{B}_{i,m_i}^c) \leq \epsilon$.

We now construct a new decoder D'_i . Let $N = KL$, where L is a large integer, and let $\Delta = T/N$. The domain of \hat{D}_i is $\{Y_i(t_k)\}_{k=1}^K$. For $k = 1, \dots, K$, let n_k be the integer which satisfies

$$(n_k - 1)\Delta < t_k \leq n_k\Delta. \quad (108)$$

The domain of D'_i is $\{Y_i(n\Delta)\}_{n=1}^N$. Let

$$D'_i(\{Y_i(n\Delta)\}_{n=1}^N) = \hat{D}_i(\{Y_i(n_k\Delta)\}_{k=1}^K). \quad (109)$$

For decoder 1, $1 \leq m_1 \leq M_1$,

$$P_{m_1}(D'_1(Y_1(0, T)) \neq \hat{D}_1(Y_1(0, T))) \quad (110)$$

$$\leq P_{m_1} \left(\bigcup_{k=1}^K \{Y_1(t_k) \neq Y_1(n_k\Delta)\} \right) \quad (111)$$

$$\leq \sum_{k=1}^K P_{m_1}(Y_1(t_k) \neq Y_1(n_k\Delta)) \quad (112)$$

$$= \sum_{k=1}^K \left[1 - \exp \left\{ - \int_{t_k}^{n_k\Delta} (aX_1(t) + bX_2(t) + \lambda_1) dt \right\} \right] \quad (113)$$

$$\leq K(aA_1 + bA_2 + \lambda_1)\Delta = (aA_1 + bA_2 + \lambda_1)T/L, \quad (114)$$

which can be made arbitrarily small as $L \rightarrow \infty$. The same holds for receiver 2. As the result, part a) of the theorem is proved.

Now, assume the decoders satisfy part a), which implies that decoder $D_i(Y_i(0, T))$ depends only on $\{Y_i(n\Delta)\}$, $1 \leq n \leq N$, or alternatively on $\{\hat{Y}_{i,n}\}$. We note that the probability law defining $\{\hat{Y}_{i,n}\}_{n=1}^N$ when $X_{i,m_i}(t)$, $i = 1, 2$ are transmitted is unchanged if $X_{i,m_i}(t)$ are replaced by $\tilde{X}_{i,m_i}(t)$ such that

$$\int_{(n-1)\Delta}^{n\Delta} X_{i,m_i}(t) dt = \int_{(n-1)\Delta}^{n\Delta} \tilde{X}_{i,m_i}(t) dt, \text{ for } 1 \leq n \leq N. \quad (115)$$

For $1 \leq m_i \leq M_i$, $1 \leq n \leq N$, $i = 1, 2$, we define

$$t_{m_i,n} = (n-1)\Delta + \frac{1}{A_i} \int_{(n-1)\Delta}^{n\Delta} X_{i,m_i}(t) dt, \quad (116)$$

and, for $1 \leq m_i \leq M_i$, let

$$\tilde{X}_{i,m_i}(t) = \begin{cases} A_i, & (n-1)\Delta \leq t < t_{m_i,n} \\ 0, & t_{m_i,n} < t < n\Delta \end{cases}. \quad (117)$$

Since the code $\tilde{X}_{i,m_i}(t)$ satisfies the condition (115), using the same decoder, the error probabilities at the receivers remain the same. However, $\tilde{X}_{i,m_i}(t)$ do not make transitions on an evenly spaced grid. Let $N' = NL$, where L is a large integer, and let $\Delta' = T/N'$. For $1 \leq m_i \leq M_i$, $1 \leq n' \leq N'$, if $\tilde{X}_{i,m_i}(t)$ is constant on $((n'-1)\Delta', n'\Delta']$, then $\hat{X}_{i,m_i}(t) = \tilde{X}_{i,m_i}(t)$ for t in that

interval. Otherwise, $\hat{X}_{i,m_i}(t) = 0$ for $t \in ((n' - 1)\Delta', n'\Delta']$. Note that \hat{X}_{i,m_i} differs from \tilde{X}_{i,m_i} for at most N intervals. Hence, $a\hat{X}_{1,m_1} + b\hat{X}_{2,m_2}$ differs from $a\tilde{X}_{1,m_1} + b\tilde{X}_{2,m_2}$ for at most $2N$ intervals. Thus, for any m_1 and m_2 ,

$$\|a\hat{X}_{1,m_1} + b\hat{X}_{2,m_2} - (a\tilde{X}_{1,m_1} + b\tilde{X}_{2,m_2})\|_1 \quad (118)$$

$$\triangleq \int_0^T |a\hat{X}_{1,m_1} + b\hat{X}_{2,m_2} - (a\tilde{X}_{1,m_1} + b\tilde{X}_{2,m_2})| \quad (119)$$

$$\leq 2N(aA_1 + bA_2)\Delta' = 2(aA_1 + bA_2)T/L \rightarrow 0. \quad (120)$$

Similarly, we have

$$\|c\hat{X}_{1,m_1} + d\hat{X}_{2,m_2} - (c\tilde{X}_{1,m_1} + d\tilde{X}_{2,m_2})\|_1 \leq 2(cA_1 + dA_2)T/L \rightarrow 0. \quad (121)$$

Let $\hat{\mu}_i$ be the error probability corresponding to the code $\{\hat{X}_{i,m_i}\}$. Then

$$|\mu_1 - \hat{\mu}_1| \leq \frac{1}{|\mathcal{M}_1|} \sum_{m_1=1}^{|\mathcal{M}_1|} \frac{1}{|\mathcal{M}_2|} \sum_{m_2=1}^{|\mathcal{M}_2|} |P(B_{1,m_1}|\tilde{X}_{1,m_1}, \tilde{X}_{2,m_2}) - P(B_{1,m_1}|\hat{X}_{1,m_1}, \hat{X}_{2,m_2})|. \quad (122)$$

Following from Lemma 2.2 of [2], for any $\epsilon > 0$, we have

$$|P(B_{1,m_1}|\tilde{X}_{1,m_1}, \tilde{X}_{2,m_2}) - P(B_{1,m_1}|\hat{X}_{1,m_1}, \hat{X}_{2,m_2})| \leq \epsilon$$

for sufficiently large L as long as (118) is satisfied. Thus part b) of the theorem is established.

Finally, following the same steps that of [2], one can show that the probability that there exists some intervals observing more than one photon can be made arbitrarily small as $\Delta \rightarrow 0$. Thus part c) of the theorem is true.

APPENDIX B

PROOF OF LEMMA 5.3

The capacity region of the MAC $(X_1, X_2) \rightarrow (Y_1, Y_1')$ is given by [6]:

$$\begin{aligned} R_1 &\leq \frac{1}{T} I(X_1(0, T); Y_1(0, T), Y_1'(0, T) | X_2(0, T)), \\ R_2 &\leq \frac{1}{T} I(X_2(0, T); Y_1(0, T), Y_1'(0, T) | X_1(0, T)), \\ R_1 + R_2 &\leq \frac{1}{T} I(X_1(0, T), X_2(0, T); Y_1(0, T), Y_1'(0, T)). \end{aligned}$$

The key part of the proof is to compute the above mutual information terms. After that, we can follow the steps in [6] to get the desired results.

In this proof, we compute $I(X_1(0, T), X_2(0, T); Y_1(0, T), Y_1'(0, T))$ in detail, and the other two terms can be computed similarly.

Let $N_1(t)$ be the number of points observed from the doubly stochastic Poisson point process Y_1 before time t , and $\mathbf{T}(t) = \{T_1, \dots, T_{N_1(t)}\}$ be the corresponding ordered arrival times. Obviously, $(N_1(T), \mathbf{T}(T))$ is a complete description of $Y_1(0, T)$. Similarly, let $N_1'(t)$ be the number of points observed from the doubly stochastic Poisson point process Y_1' before time t , and $\mathbf{T}'(t) = \{T_1', \dots, T_{N_1'(t)}'\}$ be the corresponding ordered arrival times. And $(N_1'(T), \mathbf{T}'(T))$ is a complete description of $Y_1'(0, T)$. We also define $Y_\sigma(t) = \{N(t), \mathbf{T}(t), N'(t), \mathbf{T}'(t)\}$, which is a complete description of Y_1 and Y_1' .

We note that given $x_1(0, T)$ and $x_2(0, T)$, Y_1 and Y_1' are two independent Poisson point processes with the rates $\lambda_{Y_1}(t) = ax_1(t) + bx_2(t) + \lambda_1$ and $\lambda_{Y_1'}(t) = (ad/c - b)x_2(t) + \lambda_2 - c\lambda_1/a$, respectively. Hence, given $x_1(t)$ and $x_2(t)$, the sample function density of $Y_\sigma(T)$ is [16]

$$f_{Y_\sigma(T)|X_1, X_2}(y_\sigma | x_1, x_2) = \begin{cases} \exp\left(-\int_0^T \lambda_{Y_1}(t) dt\right) \exp\left(-\int_0^T \lambda_{Y_1'}(t) dt\right), & n_1(T) = 0, n_1'(T) = 0 \\ \exp\left(-\int_0^T \lambda_{Y_1}(t) dt\right) \prod_{i=1}^{n_1(T)} \lambda_{Y_1}(t_i) \exp\left(-\int_0^T \lambda_{Y_1'}(t) dt\right), & n_1(T) \geq 1, n_1'(T) = 0 \\ \exp\left(-\int_0^T \lambda_{Y_1}(t) dt\right) \exp\left(-\int_0^T \lambda_{Y_1'}(t) dt\right) \prod_{j=1}^{n_1'(T)} \lambda_{Y_1'}(t_j), & n_1(T) = 0, n_1'(T) \geq 1 \\ \exp\left(-\int_0^T \lambda_{Y_1}(t) dt\right) \prod_{i=1}^{n_1(T)} \lambda_{Y_1}(t_i) \exp\left(-\int_0^T \lambda_{Y_1'}(t) dt\right) \prod_{j=1}^{n_1'(T)} \lambda_{Y_1'}(t_j), & n_1(T) \geq 1, n_1'(T) \geq 1 \end{cases}$$

Now, define $\hat{\lambda}_{Y_1}(t) = \lambda_1 + \mathbb{E}\{aX_1(t) + bX_2(t) | Y_\sigma(\tau); 0 \leq \tau \leq t\}$ and $\hat{\lambda}_{Y_1'}(t) = \lambda_2 - c\lambda_1/a + \mathbb{E}\{(ad/c - b)X_2(t) | Y_\sigma(\tau); 0 \leq \tau \leq t\}$. Then following from [16] (Chap. 6), Y_1 and Y_1' are self-exciting counting processes with intensity processes $\hat{\lambda}_{Y_1}(t), 0 \leq t \leq T$ and $\hat{\lambda}_{Y_1'}(t), 0 \leq t \leq T$.

Moreover the sample density function of $Y_\sigma(T)$ is given by (see (6.224) of [16])

$$f_{Y_\sigma(T)}(y_\sigma) = \begin{cases} \exp\left(-\int_0^T \hat{\lambda}_{Y_1}(t) dt\right) \exp\left(-\int_0^T \hat{\lambda}_{Y_1'}(t) dt\right), & n_1(T) = 0, n_1'(T) = 0 \\ \exp\left(-\int_0^T \hat{\lambda}_{Y_1}(t) dt\right) \prod_{i=1}^{n_1(T)} \hat{\lambda}_{Y_1}(t_i) \exp\left(-\int_0^T \hat{\lambda}_{Y_1'}(t) dt\right), & n_1(T) \geq 1, n_1'(T) = 0 \\ \exp\left(-\int_0^T \hat{\lambda}_{Y_1}(t) dt\right) \exp\left(-\int_0^T \hat{\lambda}_{Y_1'}(t) dt\right) \prod_{j=1}^{n_1'(T)} \hat{\lambda}_{Y_1'}(t_j), & n_1(T) = 0, n_1'(T) \geq 1 \\ \exp\left(-\int_0^T \hat{\lambda}_{Y_1}(t) dt\right) \prod_{i=1}^{n_1(T)} \hat{\lambda}_{Y_1}(t_i) \exp\left(-\int_0^T \hat{\lambda}_{Y_1'}(t) dt\right) \prod_{j=1}^{n_1'(T)} \hat{\lambda}_{Y_1'}(t_j), & n_1(T) \geq 1, n_1'(T) \geq 1 \end{cases}$$

We then obtain

$$\begin{aligned}
& I(X_1(0, T), X_2(0, T); Y_1, Y'_1) \\
&= h(Y_\sigma) - h(Y_\sigma | X_1, X_2) \\
&= \mathbb{E} \left[\int_0^T (\hat{\lambda}_{Y_1}(t) - \lambda_{Y_1}(t)) dt + \int_0^T (\hat{\lambda}_{Y'_1}(t) - \lambda_{Y'_1}(t)) dt \right] \\
&+ \mathbb{E} \left[\ln \left\{ \mathbf{1}(n_1(T) = 0, n'_1(T) = 0) + \mathbf{1}(n_1(T) \geq 1, n'_1(T) = 0) \prod_{i=1}^{n_1(T)} \lambda_{Y_1}(t_i) \right. \right. \\
&\quad \left. \left. + \mathbf{1}(n_1(T) = 0, n'_1(T) \geq 1) \prod_{j=1}^{n'_1(T)} \lambda_{Y'_1}(t_j) + \mathbf{1}(n_1(T) \geq 1, n'_1(T) \geq 1) \prod_{i=1}^{n_1(T)} \lambda_{Y_1}(t_i) \prod_{j=1}^{n'_1(T)} \lambda_{Y'_1}(t_j) \right\} \right] \\
&- \mathbb{E} \left[\ln \left\{ \mathbf{1}(n_1(T) = 0, n'_1(T) = 0) + \mathbf{1}(n_1(T) \geq 1, n'_1(T) = 0) \prod_{i=1}^{n_1(T)} \hat{\lambda}_{Y_1}(t_i) + \right. \right. \\
&\quad \left. \left. \mathbf{1}(n_1(T) = 0, n'_1(T) \geq 1) \prod_{j=1}^{n'_1(T)} \hat{\lambda}_{Y'_1}(t_j) + \mathbf{1}(n_1(T) \geq 1, n'_1(T) \geq 1) \prod_{i=1}^{n_1(T)} \hat{\lambda}_{Y_1}(t_i) \prod_{j=1}^{n'_1(T)} \hat{\lambda}_{Y'_1}(t_j) \right\} \right] \\
&\stackrel{(a)}{=} \mathbb{E} \left[\mathbb{E} \left[\ln \left\{ \mathbf{1}(n_1(T) = 0, n'_1(T) = 0) + \mathbf{1}(n_1(T) \geq 1, n'_1(T) = 0) \prod_{i=1}^{n_1(T)} \lambda_{Y_1}(t_i) \right. \right. \right. \\
&\quad \left. \left. + \mathbf{1}(n_1(T) = 0, n'_1(T) \geq 1) \prod_{j=1}^{n'_1(T)} \lambda_{Y'_1}(t_j) \right. \right. \\
&\quad \left. \left. + \mathbf{1}(n_1(T) \geq 1, n'_1(T) \geq 1) \prod_{i=1}^{n_1(T)} \lambda_{Y_1}(t_i) \prod_{j=1}^{n'_1(T)} \lambda_{Y'_1}(t_j) \right\} \middle| N_1(T), N'_1(T) \right] \\
&- \mathbb{E} \left[\mathbb{E} \left[\ln \left\{ \mathbf{1}(n_1(T) = 0, n'_1(T) = 0) + \mathbf{1}(n_1(T) \geq 1, n'_1(T) = 0) \prod_{i=1}^{n_1(T)} \hat{\lambda}_{Y_1}(t_i) + \right. \right. \right. \\
&\quad \left. \left. \mathbf{1}(n_1(T) = 0, n'_1(T) \geq 1) \prod_{j=1}^{n'_1(T)} \hat{\lambda}_{Y'_1}(t_j) \right. \right. \\
&\quad \left. \left. + \mathbf{1}(n_1(T) \geq 1, n'_1(T) \geq 1) \prod_{i=1}^{n_1(T)} \hat{\lambda}_{Y_1}(t_i) \prod_{j=1}^{n'_1(T)} \hat{\lambda}_{Y'_1}(t_j) \right\} \middle| N_1(T), N'_1(T) \right] \cdot (*)
\end{aligned}$$

In the above equation, step (a) follows because

$$\begin{aligned}
& \mathbb{E} \left[\int_0^T (\hat{\lambda}_{Y_1}(t) - \lambda_{Y_1}(t)) dt + \int_0^T (\hat{\lambda}_{Y'_1}(t) - \lambda_{Y'_1}(t)) dt \right] \\
&\stackrel{(b)}{=} \int_0^T \mathbb{E}[\hat{\lambda}_{Y_1}(t) - \lambda_{Y_1}(t)] dt + \int_0^T \mathbb{E}[\hat{\lambda}_{Y'_1}(t) - \lambda_{Y'_1}(t)] dt \\
&\stackrel{(c)}{=} 0,
\end{aligned} \tag{123}$$

where (b) follows because λ_{Y_1} , $\hat{\lambda}_{Y_1}$, $\lambda_{Y'_1}$ and $\hat{\lambda}_{Y'_1}$ are bounded, and hence the integral and expectation can be exchanged, and (c) follows because $\mathbb{E}[\lambda_{Y_1}(t)] = \mathbb{E}[\hat{\lambda}_{Y_1}(t)]$ and $\mathbb{E}[\lambda_{Y'_1}(t)] = \mathbb{E}[\hat{\lambda}_{Y'_1}(t)]$.

We proceed to compute (*) and obtain

$$\begin{aligned}
& I(X_1(0, T), X_2(0, T); Y_1, Y'_1) \\
&= \mathbb{E} \left[\mathbf{1}(n_1(T) \geq 1, n'_1(T) = 0) \mathbb{E} \left[\sum_{i=1}^{N_1(T)} \left(\ln \lambda_{Y_1}(t_i) - \ln \hat{\lambda}_{Y_1}(t_i) \right) \middle| N_1(T) \right] \right] \\
&+ \mathbb{E} \left[\mathbf{1}(n_1(T) = 0, n'_1(T) \geq 1) \mathbb{E} \left[\sum_{j=1}^{N'_1(T)} \left(\ln \lambda_{Y'_1}(t_j) - \ln \hat{\lambda}_{Y'_1}(t_j) \right) \middle| N'_1(T) \right] \right] \\
&+ \mathbb{E} \left[\mathbf{1}(n_1(T) \geq 1, n'_1(T) \geq 1) \mathbb{E} \left[\sum_{i=1}^{N_1(T)} \left(\ln \lambda_{Y_1}(t_i) - \ln \hat{\lambda}_{Y_1}(t_i) \right) \middle| N_1(T) \right] \right] \\
&+ \mathbb{E} \left[\mathbf{1}(n_1(T) \geq 1, n'_1(T) \geq 1) \mathbb{E} \left[\sum_{j=1}^{N'_1(T)} \left(\ln \lambda_{Y'_1}(t_j) - \ln \hat{\lambda}_{Y'_1}(t_j) \right) \middle| N'_1(T) \right] \right] \\
&= \mathbb{E} \left[\mathbf{1}(n_1(T) \geq 1) \mathbb{E} \left[\sum_{i=1}^{N_1(T)} \left(\ln \lambda_{Y_1}(t_i) - \ln \hat{\lambda}_{Y_1}(t_i) \right) \middle| N_1(T) \right] \right] \\
&+ \mathbb{E} \left[\mathbf{1}(n'_1(T) \geq 1) \mathbb{E} \left[\sum_{j=1}^{N'_1(T)} \left(\ln \lambda_{Y'_1}(t_j) - \ln \hat{\lambda}_{Y'_1}(t_j) \right) \middle| N'_1(T) \right] \right] \\
&= \mathbb{E} \left[\mathbb{E} \left[\mathbf{1}(N_1(T) \geq 1) \sum_{i=1}^{N_1(T)} \left(\ln \lambda_{Y_1}(t_i) - \ln \hat{\lambda}_{Y_1}(t_i) \right) \middle| N_1(T) \right] \right] \\
&+ \mathbb{E} \left[\mathbb{E} \left[\mathbf{1}(N'_1(T) \geq 1) \sum_{i=1}^{N'_1(T)} \left(\ln \lambda_{Y'_1}(t_i) - \ln \hat{\lambda}_{Y'_1}(t_i) \right) \middle| N'_1(T) \right] \right]. \tag{124}
\end{aligned}$$

Using Proposition 1 of [18], (124) is equal to

$$\begin{aligned}
I(X_1(0, T), X_2(0, T); Y_1, Y'_1) &= \mathbb{E} \int_0^T (\lambda_1(t) \ln \lambda_1(t) - \lambda_1(t) \ln \hat{\lambda}_1(t)) dt \\
&+ \mathbb{E} \int_0^T (\lambda'_1(t) \ln \lambda'_1(t) - \lambda'_1(t) \ln \hat{\lambda}'_1(t)) dt. \tag{125}
\end{aligned}$$

We can now follow the same argument as that in [6] and conclude that binary input is optimal.

Thus, the capacity region of the MAC $(X_1, X_2) \rightarrow (Y_1, Y'_1)$ is given by the Lemma 5.3.

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